

A SIMPLE THEORETICAL MODEL FOR RECTANGULAR MICROSTRIP RESONATOR WITH STUBS

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ABSTRACT: *In this work, we present a simple theoretical analysis of a compact rectangular microstrip resonator with stubs. The analytical model is based on a recursive algorithm for a transmission line loaded with capacitive stubs. We calculate the input impedance and the resonant frequency of this resonator and analyse the dependence of these parameters on the number and dimensions of the stubs. The numerical results are compared with those obtained by other methods and with experimental data.*

Key words: *Microstrip resonator with stubs, recursive algorithm, transmission line, equivalent capacitances.*

I. INTRODUCTION

One of the possible methods to decrease the dimensions of the microstrip resonators is the utilization of periodic stubs on the periphery of the resonators [1-4]. This method allows one a significant reduction (up to 50%) of the dimensions of the resonators and easy feasibility of the stubs of various shapes in different resonator geometries. Many different numerical approaches can be used for calculation of such resonators. In the case of the resonators with regular periodic stubs, two analytical models were used for the analysis: the model of resonant cavity with impedance boundary conditions [1], and the model of a transmission line with inductive loadings [3].

In this work, we present another simple model for analysis of the resonator of rectangular shape. In contrast to the paper [3], where the resonator is modeled by a transmission line with inductive slots, our model consists of a transmission line with stubs. The stubs play a role of capacitive elements loading the line along its length. This model can be considered as an alternative to the model with inductive slots.

Using a recursive algorithm, we calculate the input impedance and the resonant frequency of this resonator for different number of the stubs with different dimensions. The obtained results are compared with data existent in the literature. Besides, we also discuss briefly a matching design of the resonators with a feeding microstrip line.

II. THEORETICAL ANALYSIS

A. Equivalent Model of the Resonator With Stubs. The resonator with the lowest fundamental mode shown in Fig.1 can be considered as a transmission line of the width $W' = W - 2l$ loaded (not necessarily periodically) along its length L by parallel lumped capacitive elements. These capacitances simulate the effect of the lateral stubs. Fig. 2 shows a microstrip line segment with one stub and its equivalent model, where Z_c is the line characteristic impedance and C_s is the capacitance of one stub.

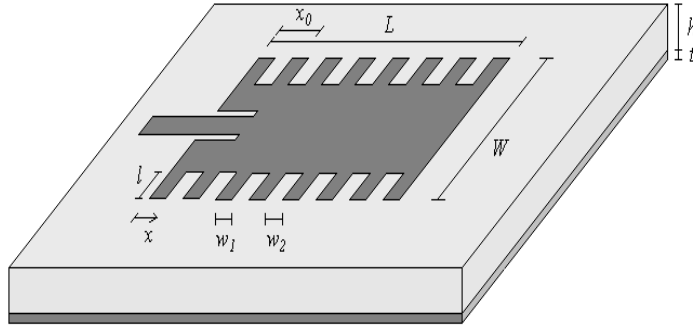


Figure 1 Rectangular microstrip resonator with equal periodical stubs.

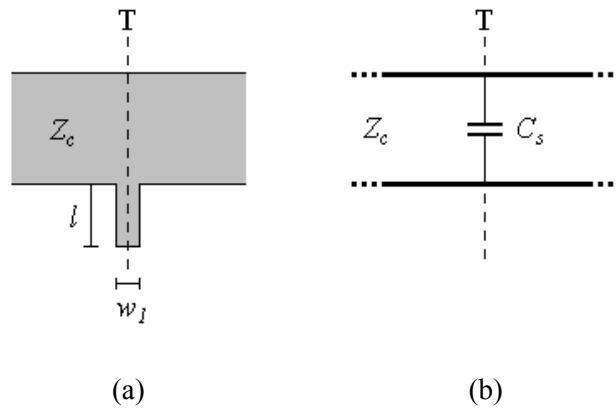


Figure 2 Microstrip line segment with one lateral stub. (a) patch geometry. (b) equivalent transmission line model.

The equivalent capacitance C_s can be calculated by different methods. Here, we consider two simplest analytical ones. The first of them is direct calculation of the capacitance by quasistatic approximation. The second method is to consider a stub as an equivalent transmission line open at its end. The following relation gives the capacitance by the first method:

$$C_s = \frac{\epsilon_{efs} \epsilon_0 w_1 l}{h} \quad (1)$$

where ϵ_0 is the free space permittivity, w_1 the stub width, l the stub length, h is the substrate height. The parameter ϵ_{efs} is the effective dielectric constant of the substrate of the microstrip line segment which forms the stub. The value of ϵ_{efs} can be calculated by (8) or (9) given in Appendix replacing W by w_1 .

According to the second method, the capacitance of the stub is calculated by

$$C_s = (1/j2\pi f Z_{cs}) \tan(k_s l) \quad (2)$$

where the characteristic impedance of the stub Z_{cs} is obtained replacing W by w_l in (6), (7), (8) and (9), $k_s=k_0(\epsilon_{efs})^{1/2}$ is the propagation constant along the stub, $k_0=2\pi f(\epsilon_0\mu_0)^{1/2}$ is the free space propagation constant, f the frequency and μ_0 the free space permeability. Notice that the both approximate methods do not take into account the possible coupling between adjacent stubs (Fig. 1) which can exist due to the fringing fields.

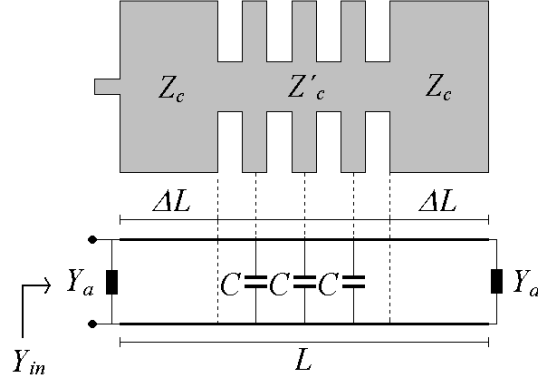


Figure 3 Equivalent transmission line model for the rectangular resonator with periodic stubs, $N_s=3$.

In Fig. 3, an equivalent transmission line model for the resonator with periodic stubs is shown. In this figure, N_s is the number of the pairs of the stubs ($N=N_s+1$ is the number of the pairs of the slots), $\Delta L=[L-Nw_2-(N-1)w_l]/2$ is the length of the two line segments with the characteristic impedance Z_c , (see formulas for Z_c in Appendix), $(L-2\times\Delta L)$ is the length of the line segment with the characteristic impedance Z'_c . The value of Z'_c can be calculated replacing W by W' in (6), (7), (8) and (9). In certain points of the line with characteristic impedance Z'_c , two stubs (one on each side of the line) are connected in parallel. Therefore, the value of each capacitance in the equivalent circuit is $C=2C_s$. C_s can be calculated by (1) or (2).

B. Input Admittance. The input admittance of the resonator at any point of the equivalent transmission line can be calculated using the following recurrent relation:

$$Y_n = Y_C \frac{Y_{n-1} + jY_C \tan(k\Delta l_n)}{Y_C + jY_{n-1} \tan(k\Delta l_n)}, \quad n=1, 3, \dots, (N+2) \quad (3)$$

where $k=k_0(\epsilon_{ef})^{1/2}$ or $k=k_0(\epsilon'_{ef})^{1/2}$ is the line propagation constant, ϵ'_{ef} is the effective dielectric constant of the microstrip line of width W' , $Y_C=1/Z_c$ or $Y'_C=1/Z'_c$ the line characteristic admittance, Y_{n-1} is the $(n-1)$ th input admittance and Δl_n is the length of the segments of the equivalent line. Finally, the input admittance Y_{in} of the resonator on the whole is

$$Y_{in}=Y_{N+2}. \quad (4)$$

C. Impedance Match. One of the possible feeds of the resonator is shown in Fig. 1. Here, it is the so-called recessed microstrip-line feeding. For the feeding line with the characteristic impedance Z_l , its optimum position x_0 along the resonator is defined by equation

$$x_0 = \frac{L}{\pi} \cos^{-1} \left(\sqrt{\frac{Z_l}{R_{in0}}} \right) \quad (5)$$

where R_{in0} is the input resistance of the nonmatched resonator at the resonant frequency [5]. At the point x_0 , the resonator is ideally matched with the input line. It will be shown below that R_{in0} depends on the number and dimensions of the stubs. Therefore, the optimum position x_0 will also depend on these parameters.

III. NUMERICAL RESULTS AND DISCUSSION

A. Input Impedance and Resonant Frequency. With a view to comparison, the input impedances $Z_{in}=R_{in}+jX_{in}$ of two rectangular resonators were computed. One of them is the conventional resonator without stubs and the other has equal periodical stubs (Fig. 5). The dimensions of the first resonator are $W=3.5\text{cm}$, $L=7.5\text{cm}$, $h=1.55\text{mm}$, and of the second one are $W=3.5\text{cm}$, $L=7.5\text{cm}$, $h=1.55\text{mm}$, $l=1.25\text{cm}$, $N=25$, $l/W'=1.25$, $K=w_1/(w_1+w_2)=0.5$ and $w_1=w_2=1\text{mm}$. The substrate permittivity of the two resonators is $\epsilon_R=4.4$. Fig. 4 shows the calculated input impedances for these two resonators.

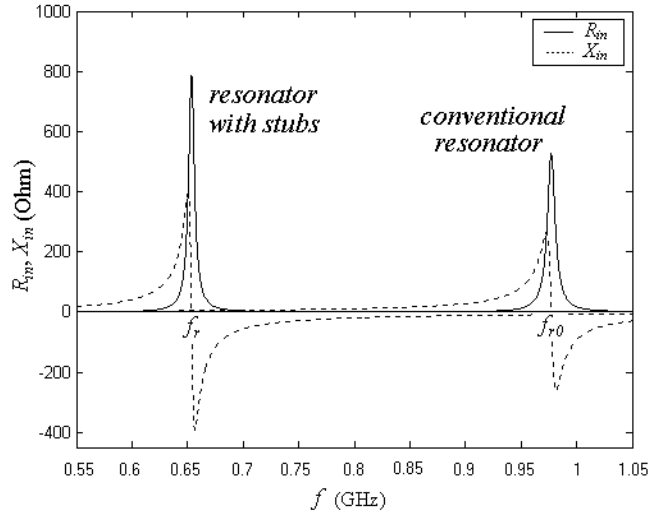


Figure 4 Input impedance of the resonator with stubs and of the conventional resonator without stubs.

We can observe in this figure a significant reduction of the resonant frequency for the resonator with stubs, a reduction of its bandwidth, and an increase of the resonant resistance.

In Table 1 below, we make a comparison of calculations of the resonant frequency (f_{r0} is the resonant frequency for the conventional resonator without stubs, f_r for the resonator with stubs) by different models and that obtained by experiments (the theoretical data for the resonator with inductive slots were taken from Fig. 5 of [3]). One can see a good agreement of our results with the data existing in the literature.

In Fig. 5, we show the patch geometry and the transmission line models with inductive loadings [3] and with capacitive loadings. The resonators have the number of slot pairs $N=25$. Fig. 6 demonstrates the input resistance R_{in} versus frequency for different number N .

TABLE 1 Comparison of Results

Model	f_{r0} (GHz)	f_r (GHz)
Transmission line with inductive loadings [3]	1.01	0.677
Transmission line with capacitive loadings	0.97	0.658
Impedance boundary conditions	1.02	0.647
Experimental data [3]	1.04	0.665

The values of N are 0, 3, 6, 9, 12, 15 and 22. The case of $N=0$ represents the conventional resonator with the patch dimensions $W=3.5\text{cm}$ and $L=7.5\text{cm}$.

Fig. 7 shows variation of the resonant frequency versus N calculated by our method and obtained by other methods. We see from this figure that for this particular geometry of the resonator, the model with inductive slots [3] gives overstated values of the resonant frequencies, but our model with capacitive stubs gives underestimated values of them in comparison with the experimental results. Notice also, that for small number of the stubs N , the curves obtained by using formulas (1) and (2) for equivalent capacitance of the stubs practically coincide, but for large number of stubs the approximation (1) gives better results.

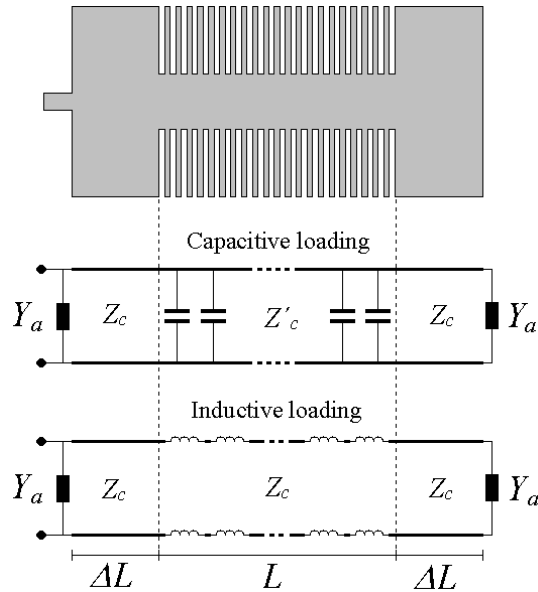


Figure 5 Patch geometry with $N=22$ slots and its transmission line equivalent models with capacitive and inductive loadings.

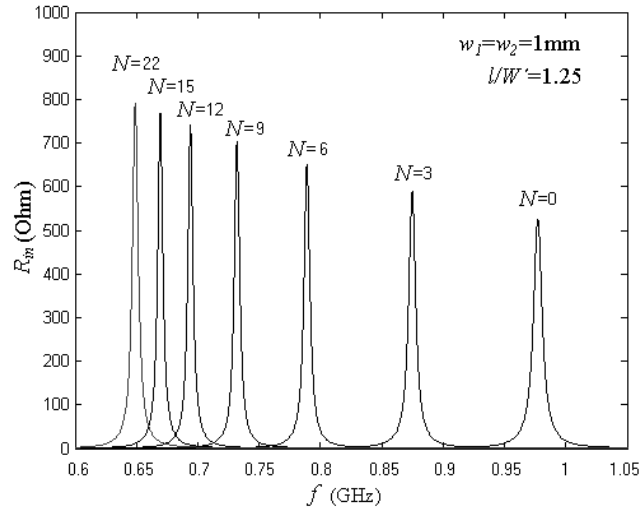


Figure 6 Input resistance R_{in} versus frequency for different number of stubs N .

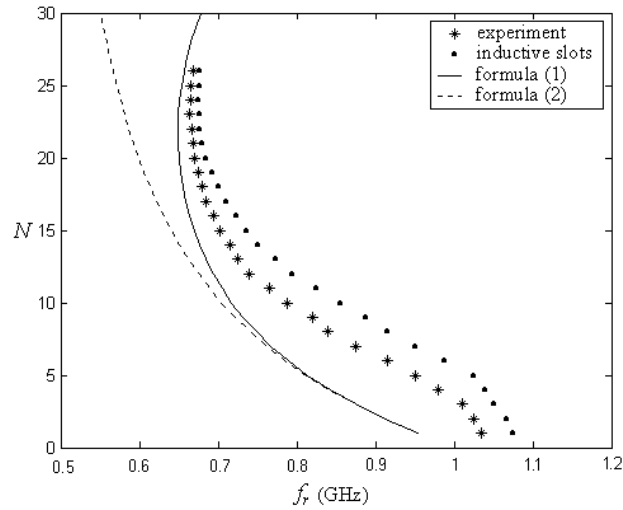


Figure 7 Resonant frequency versus N . Experimental and inductive slot model data are from [3].

Another set of data was obtained maintaining constant the following parameters: $W=3.5\text{cm}$, $L=7.5\text{cm}$, $h=1.55\text{mm}$, $N=25$ and changing the values l/W' and $K=w_1/(w_1+w_2)$. In Fig. 8, the variation of the input resistance R_{in} versus l/W' is presented. Fig. 9 shows the variation of the resonant frequency of the fundamental mode of the resonator in function of l/W' and $K=w_1/(w_1+w_2)$ calculated by different methods.

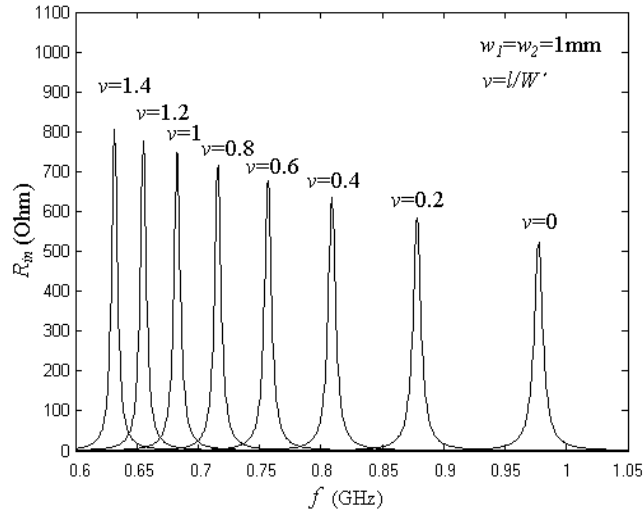


Figure 8 Input resistance R_{in} as a function of frequency for different l/W' .

Fig. 8 and Fig. 9 show that using the resonators with stubs, one can reduce the dimensions of the resonator but at the expense of its bandwidth.

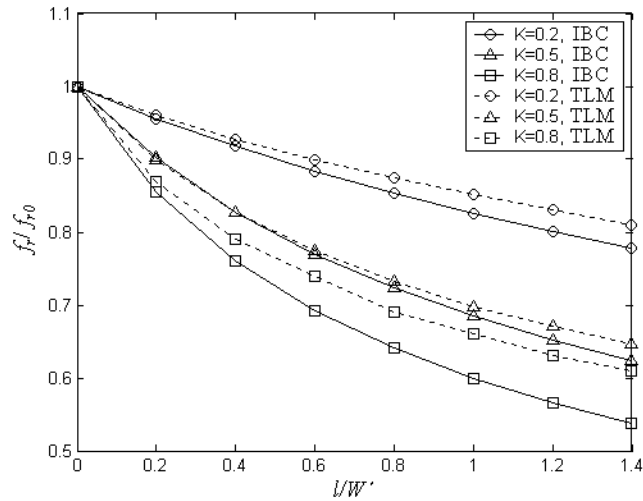


Figure 9 Normalized resonant frequency versus l/W' . IBC - Impedance Boundary Conditions, TLM - Transmission Line Model with equivalent capacitive elements.

B. Impedance Match. Using (5), we calculate the point of input feeding line connection for resonator with stubs (Fig. 1). For the characteristic impedance of the input line $Z_I=50\Omega$ and the input resistance of the resonator $R_{in0}=787\Omega$ (Fig. 4), this point is $x_0=3.14\text{cm}$. The computed by formula (1) and (3) input impedance of the resonator for the feeding point $x_0=3.14\text{cm}$ is presented on Fig. 10.

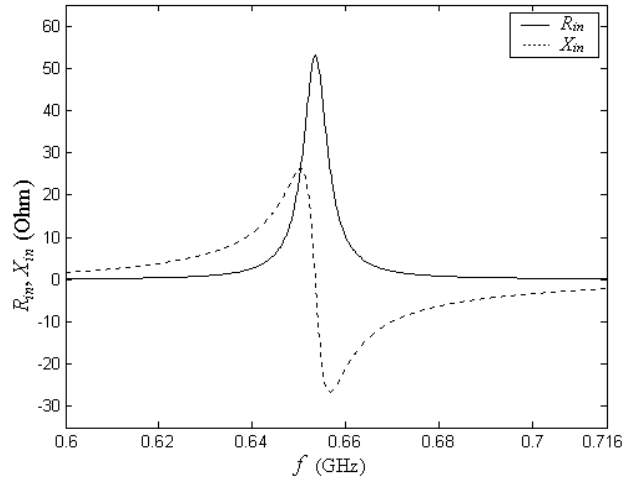


Figure 10 Input impedance of the matched resonator with equal stubs with parameters: $W=3.5\text{cm}$, $L=7.5\text{cm}$, $h=1.55\text{mm}$, $l=1.25\text{cm}$, $l/W=1.25$, $w_1=w_2=1\text{mm}$, $N=25$, $K=w_1/(w_1+w_2)=0.5$, $\epsilon_r=4.4$ and $x_0=3.14\text{cm}$.

IV. CONCLUSIONS

Compact rectangular microstrip resonator with stubs has been analyzed by the transmission line model with capacitive loadings along the line length. The reduction of the resonant frequency due to the stubs is about 35%. With appropriate choice of the number of stubs and their dimensions, 50% of this reduction can be achieved. The bandwidth of the resonators with stubs is smaller than the bandwidth of the conventional resonator without stubs. Our numerical results have good agreement with theoretical data obtained by other methods and with the experimental ones.

Finally, a remark should be made on the presented analysis. Obviously, the same resonator can be considered by two alternative transmission line models (see Fig. 5). The model with inductive slots is more accurate for the case of small width of slots w_2 and large width of stubs w_1 . On the contrary, the transmission line model with capacitive stubs is more suitable for the case of large width of slots w_2 (when we can neglect the mutual coupling of the stubs) and small width of stubs w_1 .

ACKNOWLEDGMENT

This work was supported by the Brazilian agency CNPq.

APPENDIX

For convenience, we write down here some known expressions for the parameters of microstrip lines. The characteristic impedance Z_c of a microstrip line is given by [6]:

$$Z_c = \frac{60}{\sqrt{\epsilon_{ef}}} \ln \left[\frac{8h}{W} + \frac{W}{4h} \right], \quad \frac{W}{h} \leq 1 \quad (6)$$

$$Z_c = \frac{120\pi}{\sqrt{\epsilon_{ef}} \left[\frac{W}{h} + 1.393 + 0.667 \ln \left(\frac{W}{h} + 1.444 \right) \right]}, \quad \frac{W}{h} > 1 \quad (7)$$

where ϵ_{ef} is the effective dielectric constant defined by equations:

$$\epsilon_{ef} = \frac{\epsilon_R + 1}{2} + \frac{\epsilon_R - 1}{2} \left[\left(1 + 12 \frac{h}{W} \right)^{-1/2} + 0.04 \left(1 - \frac{W}{h} \right)^2 \right], \quad \frac{W}{h} \leq 1 \quad (8)$$

$$\epsilon_{ef} = \frac{\epsilon_R + 1}{2} + \frac{\epsilon_R - 1}{2} \left[1 + 12 \frac{h}{W} \right]^{-1/2}, \quad \frac{W}{h} > 1 \quad (9)$$

the other parameters of the line are defined in the main text.

The equivalent admittance of the terminal aperture of a microstrip line is $Y_a = G + jB$ where G and B are [5]:

$$G = \frac{W}{120\lambda_0} \left[1 - \frac{1}{24} (k_0 h)^2 \right], \quad \frac{h}{\lambda_0} < \frac{1}{10} \quad (10)$$

$$B = \frac{W}{120\lambda_0} [1 - 0.636 \ln(k_0 h)], \quad \frac{h}{\lambda_0} < \frac{1}{10} \quad (11)$$

λ_0 is the free space wavelength.

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