Design of Thin Film Filters Using Differential Evolution Optimization Technique

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Abstract— **This paper presents an approach based on differential evolution (DE) for the design of thin film filters. Four projects of thin film filters are evaluated and the results are compared with particle swarm optimization (PSO). The results obtained show that DE can be a viable technique on the design of this kind of optical filters.**

Keywords - optical filters; thin film; optimization; differential evolution.

I. INTRODUCTION

Thin film filters are important components in the field of optics. There is a wide range of applications for them, like antireflective coatings, wavelength interrogation systems in optical sensors and production of polarized light in liquid crystal monitors. In telecommunications, such filters can be used as couplers, in high-capacity WDM systems and sensoring, among many others applications.

Essentially, thin films filters are made of layers sequences (usually compounded of dielectric materials) with different thickness. The careful choice of these materials, the number of layers and their thickness will determine the filter's optimum behavior in function of wavelength, incidence angle and kind of light polarization [2].

Although a thin film filter can be idealized in a way that each layer is constituted of an arbitrary materials number, for practical reasons, it is often desirable to design them using a limited number of materials. A common approach is to use two alternated materials having different refractive indices [3][4].

Analytical (classic) methods of design can be used for simple implementations. However the more complex is the optical filter, the more complex is the mathematical model that describes it. This prevents the successful application of analytical methods in many cases. This limitation makes necessary the use of other kinds of tools, like optimization techniques.

A widespread branch of the optimization nowadays is the evolutionary techniques. These techniques employ a direct search where a set of possible solutions to the problem are simultaneously evolved through variations in their parameters and stochastic decision rules that select the best ones at each iteration. In this way, they are well-suited where analytical methods do not apply or for discontinuous, non-linear and multi-modal problems. Evolutionary optimization techniques have been successfully applied in the design of optical components [5][6]. In special, genetic algorithms (GA) [3] and particle swarm optimization (PSO) [1] are used on the design of optical filters.

Among the most recent evolutionary optimization techniques, differential evolution (DE) has emerged as one of the most versatile techniques in recent years. DE is a simple and robust algorithm that has presented good performance in a great number of different areas in engineering like neural networks, synthesis of modulators and aerodynamics, for instance [7][8].

In this paper, DE is used to design thin film filters. The results obtained applying DE are compared to the results presented in [1], where PSO is employed as the optimization technique. From the best of authors' knowledge, DE was never applied to the design of thin films filters in spite of its good performance in many other areas of engineering. Additionally, previous works [9][10] have presented comparisons between DE and other evolutionary techniques in numerical benchmark problems, where DE has outperformed the others in most of the analyzed cases.

The remaining of this paper is organized as follows. Section II defines the problem to be addressed and how the optimization process deals with it. Section III provides a background about the differential evolution optimization technique. Section IV describes the thin films projects evaluated in this paper. In Section V, the results obtained are presented and analyzed while Section VI provides a summary and the conclusions.

II. OPTICAL FILTER ANALYSIS

Thin film filters can be understood as a set of N different materials films (layers) superimposed, as shown in Figure 1. Each layer j has refraction index n_i and thickness Δz_j. In this way, the design of these structures basically consists in looking for the combination of refraction indices and thickness for each film that makes the reflectance spectrum as close as possible from the desired target.

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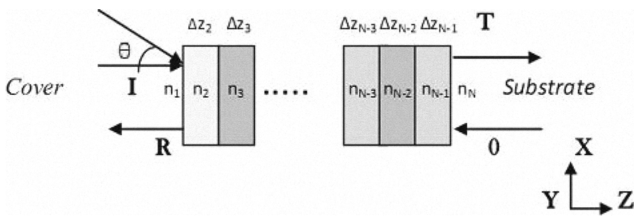

Figure 1. Thin film filter structure with N layers, including cover and substrate. I is incident beam of light and θ is angle of incidence. R is the reflected light while T is the transmitted light.

From the optimization point-of-view, the problem described above can be stated as follows:

$$x = (n_2, \Delta z_2, n_3, \Delta z_3, \ldots, n_{N-1}, \Delta z_{N-1}) \tag{1}$$

$$F(x) = \frac{1}{N_A} \sum_{n=1}^{N_A} \left[\Gamma(\lambda_i, x) - \Gamma_D(\lambda_i) \right]^2, \tag{2}$$

where:

F(x) is the objective function to be optimized;

x is a solution vector that represents a point in the domain of *F(x)*;

N_A is the number of samples used to define the reflectance curve, which is a function of wavelength;

i is the index of the sample;

$\Gamma(\lambda_i, x)$ is the reflectance value obtained in the wavelength λ_i, taking into account the given vector *x;*

$\Gamma_D(\lambda_i)$ is the filter desired reflectance in the wavelength λ_i.

To be fair when comparing results between DE and PSO, the objective function *F(x)* has to be the same that is used in [1], which is the inverse of the mean square error. However, since DE essentially deals with minimization problems, the objective function used here is the multiplicative inverse (i. e. the mean square error).

It is also important to point out that the thin film filter analysis method used to obtain $\Gamma(\lambda_i, x)$ was the characteristic matrix method [11][12]. Another important point is that the number of samples used to define the reflectance curve was fixed in one hundred ($N_A = 100$).

Each solution vector *x* corresponds to a possible thin film filter. Therefore, the solution vector *x* that makes *F(x)* assume its minimum value corresponds to the filter which gets closer to the project specifications. In this way, from a mathematical point-of-view, the process of designing thin films filters is reduced to the problem of minimizing the function *F(x)*.

III. Differential Evolution

DE was developed by Rainer Storn and Kenneth Price in 1995 to solve Chebychev polynomial fitting problem. The DE algorithm has three main advantages; finding the true global minimum regardless of the initial parameter values, fast convergence, and the usage of few control parameters [13][14]. The overall structure of DE algorithm resembles that of most other population-based search methods [8]. However, the crucial idea behind DE is a new scheme of search where new individuals, called mutant, are created through the addition of weighted difference(s) between two or more population individuals to another one [13]. Since DE works with real-coding, each individual is an *n*-dimensional vector, where *n* is the number of parameters to be optimized.

The main operators associated to DE are:

- **Mutation**: main DE operator. Consists in create, for each individual of the current population, a new associated individual, called mutant, from a weighted sum of two or more other individuals.
- **Recombination (crossover)**: consists in promoting parameters exchange between a parent individual and its corresponding mutant, resulting in a trial individual. DE uses a non-uniform crossover that can take parameters from one parent individual more often than it does from others.
- **Selection**: operator that selects the healthiest (with the best fitness) between a parent individual and its corresponding trial individual.

As can be noticed, DE algorithm uses mutation operator as a search mechanism and selection operator to direct the search towards the prospective regions in the search space. This extraction of distance and direction information from the population to generate random deviations results in an adaptive scheme with excellent convergence properties [14][15].

The control parameters of DE are the population size *NP*, the crossover constant *CR*, and the scaling factors *F* and λ, where $CR \in [0,1]$ and F and $\lambda \in [0,2]$.

The general procedure for the DE algorithm is as follows. Firstly, an initial population is uniformly generated in the search space. Then, for each individual is created a corresponding mutant. Recombination comes next, producing a trial individual for each parent individual. Finally, selection process chooses between each parent individual and its trial counterpart, based in their fitness, in order to generate a new population with the healthiest individuals. The basic DE algorithm is summarized in Figure 2.

```
Algorithm:

  Begin
        Initialization
        Evaluation
  Repeat
        Mutation
        Recombination
        Evaluation
        Selection
  Until (stop criterions are met)
```

Figure 2. DE basic algorithm.

As mutation process plays a determinant role in DE, many schemes are proposed in the literature [14]. In this paper, the employed mutation scheme is the DE/rand-to best/1. This scheme is described by

$$v = x_{i,G} + \lambda.(x_{best,G} - x_{i,G}) + F.(x_{r2,G} - x_{r3,G}), \quad (3)$$

where v is the mutant vector; $x_{i,G}$, $x_{r2,G}$, $x_{r3,G}$ are three different individuals chosen randomly; $x_{best,G}$ is the best individual of the current generation; F and λ are the scaling factors, where F controls the random displacement of $x_{i,G}$ and λ controls the displacement towards $x_{best,g}$. The subscript G indicates the generation number. The Figure 3 graphically depicts the creation process of a mutant using the scheme DE/rand-to best/1.

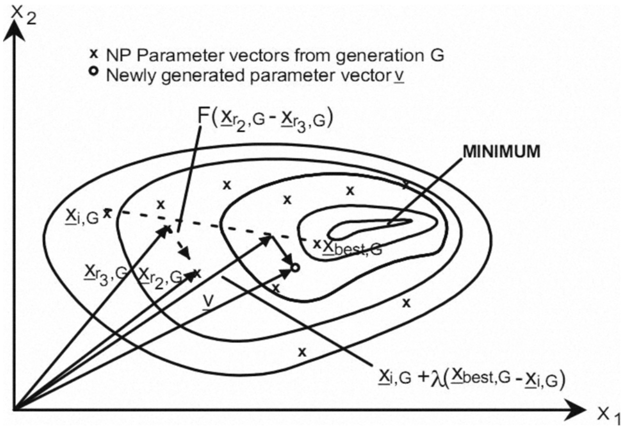

Figure 3. Two dimensional example of an objective function showing its contour lines and the generation of a mutant v from scheme DE/rand-to best/1.

It can be noted that the mutant vector v is placed between a random individual and the best individual of the current population.

The scheme DE/rand-to best/1 was chosen due to its capacity of use information from random vectors and the best individual, giving to the algorithm characteristics of randomnees and convergence, which is statistically more efficient than the other schemes.

IV. Thin Film Projects

In this section, the four thin film filters projects to be analyzed are described. The first two are proposed in [5] and the last two are proposed in [16]. These projects are also analyzed in [1] using PSO.

A. *Antireflective Coatings*

The first two projects consist in designing filters called antireflective coatings. Antireflective coatings are a type of optical coating applied to the surface of lenses and other optical devices to reduce reflection. This improves the efficiency of the system since less light is lost. These antireflective coatings are often designed to act in the spectral zone between ultraviolet and infrared.

In these two projects the number of layers is also a variable to be optimized, thus the problem becomes more complicated, because it can be said that it is a problem with a variable number of parameters to be optimized. The total number of parameters n_p is given by

$$n_p = 2.n + 1, \quad (4)$$

since two parameters, index of refraction and thickness, are associated to each layer, and the total number of layers n is also a variable to be optimized by the algorithm.

1) ***Project 1****:* The objective here is to reduce to zero the reflectance of a thin film filter with substrate index of refraction 4, in the spectral region that goes from 7.7μm to 12.3μm. The detailed specifications of this project are shown in TABLE I.

TABLE I. PROJECT 1 SPECIFICATIONS.

Specification	Value
Spectral Range (μm)	7.7 to 12.3
Angle of Incidence (degress)	0
Cover Refractive Index	1.0
Substrate Refractive Index	4.0
Refractive Indices	2.2 and 4.2
Maximum Number of Layers	4
Layer Minimum Thickness (μm)	0.01
Filter Maximum Thickness (μm)	11.2

2) ***Project 2***: In this project, the objective also consists in reducing to zero the reflectance of a filter with substrate refractive index 4. However, this time, the spectral region of interest goes from 2.8μm to 5.5μm. The detailed specifications of this project are shown in TABLE II.

TABLE II. PROJECT 2 SPECIFICATIONS.

Specification	Value
Spectral Range (μm)	2.8 to 5.5
Angle of Incidence (degrees)	0
Cover Refractive Index	1.0
Substrate Refractive Index	4.0
Refractive Indices	1.35 and 2.4
Maximum Number of Layers	8
Layer Minimum Thickness (μm)	0.01
Filter Maximum Thickness (μm)	8.0

B. *Optical Filter for Sensors Apllications*

Thin film filters can be used for interrogating wavelength in sensor systems. By using ramp-shaped reflectance spectrums, it is possible to convert quantities expressed by frequency deviation to quantities expressed in variation of optical power, which is simpler to detect [1].

1) ***Project 3***: This project consists in a filter immersed in air with a reflectance spectrum that looks like an ascending ramp in the interval from 0.4μm to 0.7μm. The minimium reflectance is 0% and the maximum is 100%. The detailed specifications of this project are shown in TABLE III.

TABLE III. PROJECT 3 SPECIFICATIONS.

Specification	Value
Spectral Range (μm)	0.4 to 0.7
Angle of Incidence (degrees)	0

Cover Refractive Index	1.0
Substrate Refractive Index	1.0
Refractive Indices	1.0583 and 1.68
Number of Layers	15
Layer Minimum Thickness (μm)	0.05
Layer Maximum Thickness (μm)	0.33

C. *Unconventional Optical Filters*

The fourth project is an unusual project of an optical filter that does not have any straight application but is very useful to test the efficiency of DE in designing complex filters.

1) ***Project 4***: This project consists in a filter immersed in air capable of reflecting 50% of the luminous energy in the interval of 0.4μm to 0.5μm (blue light range) and 100% of the luminous energy between 0.6μm and 0.7μm (green light range). In the interval of 0.5μm to 0.6μm (red range) the filter must be transparent. The detailed specifications of this project are shown in TABLE IV.

TABLE IV. PROJECT 4 SPECIFICATIONS.

SPECIFICATION	VALUE
Spectral Range (μm)	0.4 to 0.7
Angle of Incidence (degrees)	0
Cover Refractive Index	1.0
Substrate Refractive Index	1.0
Refractive Indices	1.46 and 2.1
Number of Layers	20
Layer Minimum Thickness (μm)	0.045
Layer Maximum Thickness (μm)	0.24

V. SIMULATION RESULTS AND ANALYSIS

In this section, it is presented the results obtained applying DE to the projects described in Section IV. As previously stated, the analysis presented here is carried out based on both the predefined targets and the results described in [1], where PSO is used as the optimization technique.

The results for each project are summarized in two figures and one table. The first figure shows the reflectance profile obtained using DE in comparison to the target and PSO's reflectance presented in [1], in function of the defined spectral range. Curves of error in dB are also provided in this figure. The second figure shows the layers configuration obtained using DE in comparison to that shown in [1]. The table shows the values of the objective function obtained using both DE and PSO, or in other words, the mean square reflectance error.

The values of the control parameters of DE are summarized in TABLE V.

TABLE V. VALUES OF THE CONTROL PARAMETERS USED IN THE PROJECTS.

Project	NP	*F*	λ	CR
1	100	0.4	0.3	0.5
2	100	0.4	0.3	0.5
3	80	0.2	0.3	0.5
4	80	0.2	0.3	0.5

A. *Project 1*

The filter designed using DE presents a better performance concerning the reflectance than the filter designed in [1], as can be seen in Figure 4. The mean reflectance for PSO is 0.0217 while for DE is just 0.0203, with standard deviation of 0.0056 and 0.040, respectively. This is emphasized by the values in TABLE VI. The mean square reflectance error associated to DE is around 15% less than that obtained using PSO.

In terms of filter configuration, DE provided a filter with four layers and with the double of the length achieved by PSO, as can be seen in Figure 5.

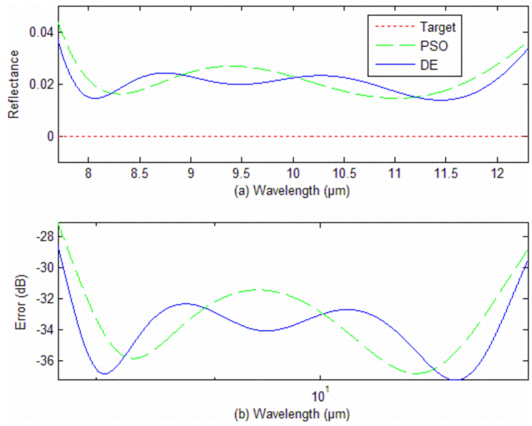

Figure 4. Project 1 (a) reflectance and (b) dB error curves.

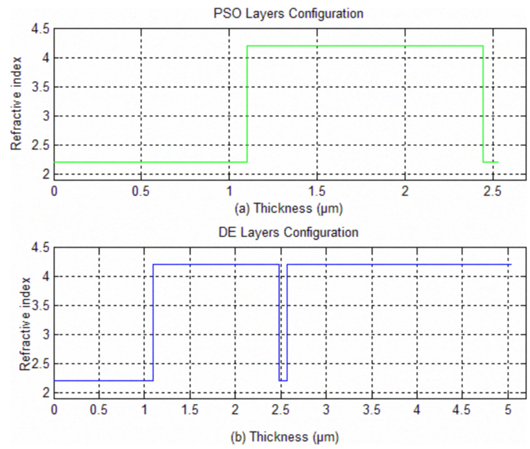

Figure 5. (a) PSO and (b) DE layers configuration obtained in project 1.

TABLE VI. OBJECTIVE FUNCTION VALUES (OFV) OBTAINED IN PROJECT 1.

Algorithm	OFV
DE	4.3725×10^{-4}
PSO	5.1387×10^{-4}

B. *Project 2*

In this project, DE obtained once again a better result, as indicated by Figure 6. DE mean square reflectance error is about 56% less than that obtained using PSO, as can be seen in

TABLE VII. In special, the mean reflectance is 0.0093 for PSO and while for DE is just 0.0058, showing the best efficiency of the last one. The standard deviations are 0.0032 and 0.030 for DE and PSO, respectively.

From Figure 7, it can be noted that the filter designed by DE is about three times longer than PSO's filter and also has three times more layers. This means that the filter designed by DE is more complex than PSO's, but once again more efficient in terms of reflectance.

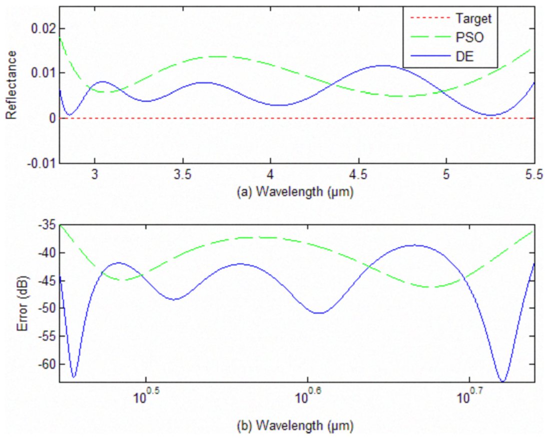

Figure 6. Project 2 (a) reflectance and (b) dB error curves.

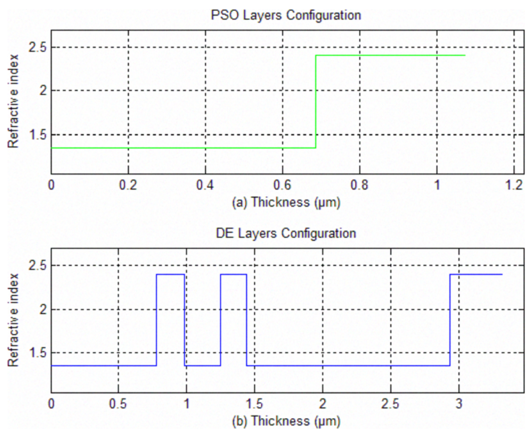

Figure 7. (a) PSO and (b) DE layers configuration obtained in project 2.

TABLE VII. OBJECTIVE FUNCTION VALUES (OFV) OBTAINED IN PROJECT 2.

Algorithm	OFV
DE	4.3085×10^{-5}
PSO	9.8232×10^{-5}

C. Project 3

In this project, the number of layers is not a variable (i.e. its value is already known). In spite of the filters designed by DE and PSO have almost the same length (Figure 9), DE mean square reflectance error is about 44% less than the achieved by PSO, as can be noted from Figure 8 and TABLE VIII. The angular coefficient of the target is 3.3333. On the other hand, the angular coefficients for results obtained by DE and PSO are 3.3069 and 3.3161, respectively, showing a slightly match for DE.

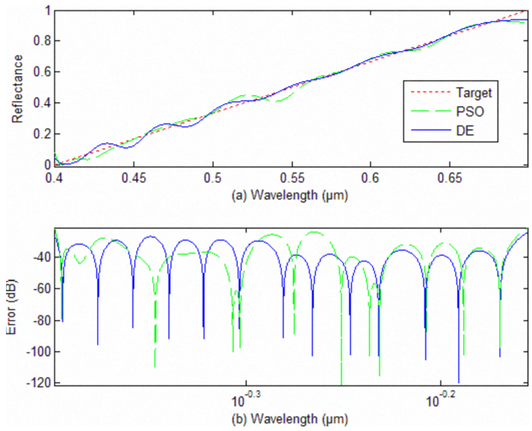

Figure 8. Project 3 (a) reflectance and (b) dB error curves.

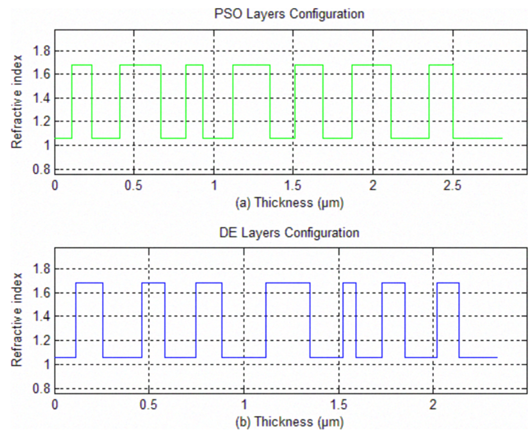

Figure 9. (a) PSO and (b) DE layers configuration obtained in project 3.

TABLE VIII. OBJECTIVE FUNCTION VALUES (OFV) OBTAINED IN PROJECT 3.

Algorithm	OFV
DE	4.0766×10^{-4}
PSO	7.3099×10^{-4}

D. Project 4

In this project, DE slightly outperformed PSO again, but both techniques obtained very similar results. The filters obtained using PSO and DE have almost the same length (see Figure 11), and DE mean square reflectance error is only about 12% less than that obtained using PSO, as can be noted in TABLE IX.

Figure 10 shows the reflectance curves obtained using DE and PSO and the associated errors.

Regarding to each range, DE has obtained better results both for the mean reflectance and standard deviation, except for the first range (blue) where PSO's mean reflectance value (0.4923) is only 1.54% apart from the target value (0.5) while

DE's is 14.7%. For the intermediary range (0.5-0.6 μm), DE presented a better result than PSO, with a more homogeneous behavior and fitting better the target. Besides, for the upper range, DE obtained an error significantly smaller than PSO's.

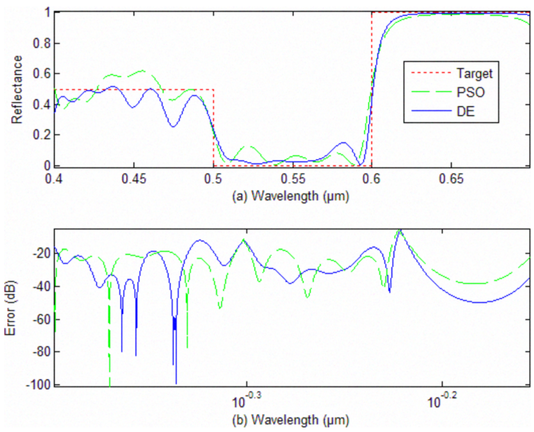

Figure 10. (a) Reflectance and (b) dB error curves of project 4.

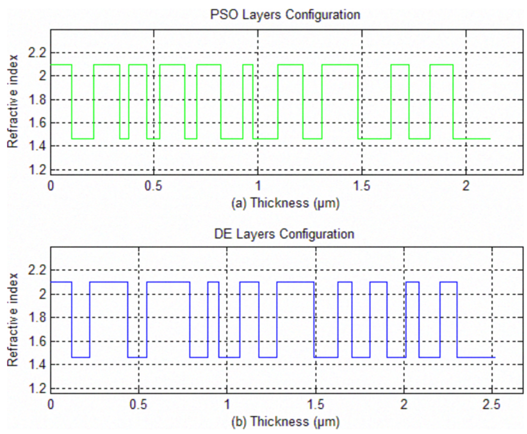

Figure 11. (a) PSO and (b) DE layers configuration obtained in project 4.

TABLE IX. OBJECTIVE FUNCTION VALUES (OFV) OBTAINED IN PROJECT 4.

Algorithm	OFV
DE	0.0089
PSO	0.0101

VI. SUMMARY AND CONCLUSIONS

This paper presented the design of four thin films filter by the usage of differential evolution optimization technique. Differential evolution is a fast, simple an easy-to-use technique, with excellent convergence properties.

DE performed better than PSO in all analyzed projects. In two of the four analyzed projects, the error provided by PSO is almost twice the error provided by DE. Additionally, in all four projects, DE designed filters with reflectance spectrums nearer to the target curve.

In general, DE has provided longer filters and with more complex configurations than those reported in [1]. However, the efficiency of the filters designed by DE is superior in all cases.

These results show that DE is a viable optimization technique for the design of thin film filters.

REFERENCES

[1] G. Borges, C. Sales, M. Souza, J. Costa, "Design of multilayer optical filters using PSO," Original in Portuguese, Proceedings of the 25th Brazilian Symposium on Telecommunications, Recife, PE, 2007.

[2] A. Chandran, "Self-Assembled multilayered dielectric spectral filters", M.S. thesis, Faculty of Virginia Polytechnic Institute, 2001.

[3] D.G. Li and A.C. Watson, "Optical thin film optimization design using genetic algorithms," IEEE International Conference on Intelligent Processing Systems, China, 1997.

[4] Ronald R. Willey, "Practical design and production of optical thin films," New York: Marcel Dekker Inc., 2002.

[5] J. A. Dobrowolski, Parmejeet Panhhi, e Martin High, " Antireflection coatings designed for two different infrared substrares", Appl. Opt. 35, 2934-2946 (1996).

[6] C. Riziotis, A.V. Vasilakos, "Computational intelligence in photonics technology and optical networks: A survey and future perspectives," Informat. Sci. (2007), doi:10.1016/j.ins.2007.06.012.

[7] R. Storn, K Price, "Differential Evolution," available at: http://www.icsi.berkeley.edu/~storn/code.html, accessed on April 2009.

[8] T. Rogalsky, R. W. Derksen and S. Kocabiyik, "Differential Evolution in aerodynamic optimization," Proceedings of the 46th Annual Conference of the Canadian Aeronautics and Space Institute, pp. 29-36, 1999.

[9] S Mishra, S. K., "Global optimization by differential evolution and particle swarm methods: evaluation on some benchmark functions," (September 30, 2006). Available at SSRN: http://ssrn.com/abstract=933827.

[10] Jakob Vesterstrøm and René Thomsen, "A comparative study of differential evolution, particle swarm optimization, and evolutionary algorithms on numerical benchmark problems," Congress on Evolutionary Computation, vol. 2, pp. 1980-1987, 2004.

[11] S. T. Peng, Theodor Tamir, e Henry L. Bertoni, "Theory of periodic dielectric waveguides," IEEE Transactions on Microwave Theory and Techniques, vol. 23, pp. 123-133, 1975.

[12] M. Born and E. Wolf, "Principles of optics," New York: Pergamon, 1987.

[13] R. Storn, K Price, "Differential Evolution – a simple and efficient heuristic for global optimization over continuous spaces," Journal of Global Optimization, vol. 11, issue 4, pp. 341-349, 1997.

[14] D. Karaboga and S. Okdem, "A simple and global *o*ptimization algorithm for *e*ngineering. *p*roblems: differential evolution algorithm," Turk J Elec Engin, vol. 12, no.1, 2004.

[15] R. Storn, "On the usage of differential evolution for function optimization," Biennial Conference of the North American Fuzzy Information Processing Society (NAFIPS), pp. 519-523, Berkley, USA, 1996.

[16] M. J. Sousa, "Optical Bragg grating design using genetic algorithms and parallel processing", Original in Portuguese, Federal University of Pará , 2003.