MDP-based Resource Allocation for Triple-Play Transmission on xDSL Systems

Lamartine V. de Souza[a], Glaucio H. S. de Carvalho[b], Diego L. Cardoso[a], Solon V. de Carvalho[c], Carlos R. L. Francês[a], João C. W. A. Costa[a] and Jaume Rius i Riu[d]

[a]Federal University of Pará (UFPA), Rua Augusto Corrêa nº 01, Guamá, Belém/PA, Brazil.
[b]Federal University of Pará (UFPA), Santarém/PA, Brazil.
[c]Brazilian Institute for Space Research (INPE), Av. dos Astronautas nº 1758, Jd. Granja, São José dos Campos/SP, Brazil.
[d]Ericsson AB, Stockholm, Sweden.

ABSTRACT

Many broadband services are based on multimedia applications, such as voice over internet protocol (VoIP), video conferencing, video on demand (VoD), and internet protocol television (IPTV). The combination "triple-play" is often used with IPTV. It simply means offering voice, video and data. IPTV and others services uses digital broadband networks such as ADSL2+ (Asymmetric Digital Subscriber Line) and VDSL (Very High Rate DSL) to transmit the data. We have formulated a MDP (Markov Decision Process) for a triple-play transmission on DSL environment. In this paper, we establish the relationship between DSL transmission characteristics and its finite-state Markov model for a triple-play transmission system. This relationship can be used for a resource management for multimedia applications delivered through a broadband infrastructure. The solution to our optimization problem can be found using dynamic programming (DP) techniques, such as value iteration and its variants. Our study results in a transmission strategy that chooses the optimal resource allocation according the triple-play traffic requirements, defined in technical report TR-126 (Triple-Play Services Quality of Experience Requirements) from DSL Forum, minimizing quality of service (QoS) violations with respect to bandwidth. Three traffic classes (video, audio, and best effort internet data) are defined and analyzed. Our simulation results show parameters like as blocking probability for each class, link utilization and optimal control policies. The MDP-based approach provides a satisfactory way of resource management for a DSL system.

Keywords: Markov decision process, triple-play, DSL systems, resource management

1. INTRODUCTION

About the provisioning of triple-play applications, there is a critical issue: how to schedule traffic and allocate bandwidth among the triple-play services [1 2 3]. That question is challenging because VoD, VoIP, and high speed internet have different QoS requirements and thus each of them deserve different treatment from networks's point of view. Thus, the proper design of resource allocation in broadband network access, also knowns as last mile networks, plays a key role. Among the current broadband network access solutions (HFC–Hybrid Fibre-Coaxial, WiMAX–Worldwide Interoperability for Microwave Access, PLC–Power Line Communications), one of the most prominent is undoubtedly DSL technologies. The reason of this is clearly: the telephone lines already spread worldwide. Thus, in current paper we focus on resource allocation in DSL networks to support triple play services. Since initial DSL solutions as ADSL are not capable of supporting these services; we assume the extended version of ADSL, the ADSL2+, which can offer 16 Mbps downstream and 800 kbps upstream [4].

The problem of optimally allocating resource in a finite capacity link offering service with different QoS requirements may be viewed as a sequential decision process. In this context, SMDP arises as outstanding solution. SMDP has been widely used in telecommunications to obtain optimal policies. For instance: Yagan et al.[5 6] present a joint bandwidth allocation and buffer management scheme using SMDP in wireless ad hoc

Further author information: (Send correspondence to Lamartine V. de Souza)
E-mail: lvsouza@ufpa.br, www.lea.ufpa.br/lamartine, Telephone: +55 91 32017740

Broadband Access Communication Technologies II, edited by Raj Jain, Benjamin B. Dingel, Shozo Komaki, Shlomo Ovadia
Proc. of SPIE Vol. 6776, 67760F, (2007) · 0277-786X/07/$18 · doi: 10.1117/12.750205

networks. Pandana et al.[7] present a transmission strategy that chooses the optimal modulation level and transmit power using MDP and reinforcement learning (RL) algorithm in point-to-point wireless communications. Yu et al.[8] formulate a QoS provisioning problem for adaptive multimedia in wireless networks as SMDP. Chang et al.[9] show that MDP approach provides a systematic way of defining call admission function and yields better networks utilization in ATM networks. Liu et al.[10] propose a dynamical packet transmission strategy for data service modeling the noisy channel by a SMDP. Kalyanasundaram et al.[11] develop two different admission control schemes to provide QoS in multiservice networks using SMDP also. In spite of its broadly application on telecommunication networks, we do not find any work on literature that aims at studying optimal policies in DSL network and, particularly, for triple play services using SMDP. Thus, in this paper we formulate the problem of resource allocation as a SMDP. We search for optimal policies considering the well-known weighted blocking and applying high priority for real time service as VoIP and VoD service classes that have more stringent QoS requirements. Due to the elastic characteristic of TCP traffic, we consider that internet service share equally the bandwidth not used by real time calls. The current version of IP protocol (IPV4) support the best-effort traffic service; nevertheless, in order to satisfy the minimum QoS requirements of internet services, a minimum bandwidth is fixed for this service. We illustrate the effectiveness of our model by means of numerical examples.

The rest of this paper is organized as follows. Next section we explain the basic concepts of DLS transmission. In section 3, we present the modeling of the resource allocation. The performance of SMDP algorithm is assessed in section 4. Finally, conclusions and possible future researches are drawn in section 5.

2. DSL

The xDSL access technologies have been developed by the telephone companies to provide high-speed data rates over regular telephone wires. The term xDSL covers a number of similar yet competing forms of DSL, including ADSL, SHDSL (Single-Pair High Speed DSL), HDSL (High-Bit-Rate DSL), and VDSL (Very High Rate DSL)[12] .

DSL services are dedicated, point-to-point, public network access over twisted-pair copper wire on the local loop (last mile) between a network service provider (NSPs) central office and the customer site, or on local loops created either intra-building or intra-campus.

Delivery of DSL services requires a copper pair configuration of a standard voice circuit with an DSL modem at each end of the line, creating three channels: a high speed downstream channel, a medium speed upstream channel, and a plain old telephone service (POTS) channel for voice. Data rates depend on several factors including the length of the copper wire, the wire gauge, presence of bridged taps, and cross-coupled interference.

Standardized in 2005, ADSL2+ is normally bundled into one upgrade to the ADSL system by the service provider. This upgrade has opened the door to new video service support capabilities and a host of new service offerings from the service providers [13] .

ADSL2+ use double the downstream bandwidth as compared to the ADSL2 transceiver defined in ITU-T Rec. G.992.3. Systems support a net data rate ranging up to a minimum of 16 Mbps downstream and 800 kbps upstream [4] . In this paper, we have used the data rate of 16 Mbps for downstream according Technical Report TR-100 from DSL Forum [14] .

3. SEMI-MARKOV DECISION PROCESS

The system under consideration consists of a link with a finite capacity of B Mbps, which is shared by the following application: video (VoD), voice (VoIP), and data. As real time services, video and voice are delay-sensitive and require a constant bandwidth in order to fulfill their QoS requirements. On the other hand, data service has less stringent requirement. As an elastic traffic, it can tolerate variations in the service rate due to the flow control mechanism of TCP. Moreover, data service share equally the bandwidth not used by video and voice calls, which means that the service rate of each data call can change over time, depending on the number of ongoing video, voice, and data calls. However, in order to satisfy the minimum QoS requirement of data service, a minimum bandwidth is fixed. Thus, if an incoming data call faces less than the minimum bandwidth, its connection request is denied.

For the sake of Markov modeling, video, voice, and data follow Poisson processes mutually independents with parameters λ_{vi}, λ_{vo}, and λ_d, respectively. The service time of video, voice, and data calls are random variables exponentially distributed with parameters $1/\mu_{vi}$, $1/\mu_{vo}$, and $1/\mu_d$, respectively. If an incoming video connection is accepted, then it will receives a fixed amount of B_{vi} bandwidth. Hence, with v_i video calls into the system, there is an amount of $v_i B_{vi}$ of the total capacity B busy. Likewise for voice calls, if an incoming voice call is accepted, then it is assigned a fixed amount of B_{vo} bandwidth. Again, with v_o voice call into the system, there is an amount of $v_o B_{vo}$ of the total capacity B busy. Data calls share the bandwidth not used by video and voice calls; thus, let $\psi = v_i B_{vi} + v_o B_{vo}$ be the total bandwidth used by real time calls. Hence with d ongoing data calls, each of then receives an amount of $\frac{B-\psi}{d}$ of the available bandwidth. With v_i, v_o, d into the system, the services completion rates of them are $v_i\mu_{vi}$, $v_o\mu_{vo}$ and $(B-\psi)\mu_d$, respectively.

Whenever an incoming call of video or voice is accepted, it will preempt the bandwidth being used by data call. Since there is a minimum bandwidth requirement for data call, it is needed to determine if the remainder bandwidth is enough to accommodate all the existing data calls. After the admission, the remainder bandwidth can support $\theta = \lfloor \frac{B-\psi}{B_d} \rfloor$ data calls with bandwidth B_d, where $\lfloor x \rfloor$ is the largest integer not greater than x. Thus, if $d < \theta$, then the system can support all the existing calls with bandwidth more than B_d; otherwise, some data calls will be preempted and dropped and the system will reduce the bandwidth of the remainder ones (θ) to B_d. Mathematically, the number of data call into the system after the admission will be given by $min(d, \theta)$.

More formally, the proposed optimal resource allocation is modeled as a SMDP, whose the state is given by:

$$\Phi = \{(v_i, v_o, d, e)/0 \leq v_i \leq \lfloor \frac{B}{B_{vi}} \rfloor, 0 \leq v_o \leq \lfloor \frac{B}{B_{vo}} \rfloor, 0 \leq d \leq \lfloor \frac{B}{B_d} \rfloor, e \in \{0, 1, 2\}\} \tag{1}$$

where v_i, v_o, and d are the random variables previously introduced. The maximum number of video, voice, and data calls are computed as $\lfloor \frac{B}{B_{vi}} \rfloor$, $\lfloor \frac{B}{B_{vo}} \rfloor$, $\lfloor \frac{B}{B_{vd}} \rfloor$, respectively. e is the last event occurred. This information is introduced in the state space in order to define the set of possible actions in each state. Accordingly the system dynamics, the values of e may be:

- An arrival of data call, departure of video, voice, and data call, $e = 0$;
- An arrival of video call, $e = 1$;
- An arrival of voice call, $e = 2$.

We assume that each state means the system's configuration just after an event occurrence and just before a decision making. The decision epochs are the arrival of video and voice calls, i.e., $e = 1, 2$. For $e = 0$ no decision is taken. Let $\sigma = B_j + \sum(v_i B_{vi} + v_o B_{vo})$ be the amount of bandwidth used by real time call plus the bandwidth required by the new connection request of type j, which is determined by the value of $e =$, *i.e.*, if $e = 1$, then $B_j = B_{vi}$ and if $e = 2$, then $B_j = B_{vo}$. When $\sigma \leq B$ and the random variable e is 1 or 2; then a reject or accept decision must be made. Each decision variable can be either 1 or 0, where 1 stands for accept and 0 for reject. On the other hand, when $\sigma > B$, only a reject decision is possible. Thus, the action space can be expressed for all $i \in \Phi$ as follows:

$$A(i) = \begin{cases} a = 1, & \text{if } e = 1, 2 \text{ and } \sigma \leq B; \\ a = 0, & \text{if } e = 0 \text{ or } \sigma > B \text{ and } e = 1, 2. \end{cases} \tag{2}$$

Here the service completion epochs and arrival of data call are defined as fictitious decision epochs in addition to real ones $(e = 1, 2)$. *By default*, the actions $a = 0$ are used in these decision epochs. For this process, given that in a decision epoch the system is in the state $i \in \Phi$ and the action $a \in A(i)$ is chosen, we define:

- $\tau_i(a)$ as the expected time until the next decision epoch;
- $p_{ij}(a)$ as the probability that in the next decision epoch the state will be j;

- $C_i(a)$ as the expected cost incurred until the next decision epoch.

These quantities may be computed as:

$$\tau_i(a) = \frac{1}{\lambda_{vi} + \lambda_{vo} + \lambda_d + v_i\mu_{vi} + v_o\mu_{vo} + (B-\psi)\mu_d}. \tag{3}$$

$$p_{ij}(a) = \begin{cases} \lambda_{vi}\tau_i(a), & \forall i = (v_i, v_o, d, 1),\ j = (v_i+1, v_o, min(d,\theta), e) \in \Phi, \\ & \text{and } a = 1 \in A(i); \\ \lambda_{vi}\tau_i(a), & \forall i = (v_i, v_o, d, 1),\ j = i \in \Phi \text{ and } a = 0 \in A(i); \\ \lambda_{vo}\tau_i(a), & \forall i = (v_i, v_o, d, 2),\ j = (v_i, v_o+1, min(d,\theta), e) \in \Phi, \\ & \text{and } a = 1 \in A(i); \\ \lambda_{vo}\tau_i(a), & \forall i = (v_i, v_o, d, 2),\ j = i \in \Phi \text{ and } a = 0 \in A(i); \\ \lambda_d\tau_i(a), & \forall i = (v_i, v_o, d, 0),\ j = (v_i, v_o, d+1, e) \in \Phi, \\ & a = 0 \in A(i), \text{ and } d < \lfloor \frac{B-\psi}{B_d} \rfloor; \\ \mu_{vi}\tau_i(a), & \forall i = (v_i, v_o, d, 0),\ j = (v_i-1, v_o, d, e) \in \Phi, \\ & \text{and } a = 0 \in A(i); \\ \mu_{vo}\tau_i(a), & \forall i = (v_i, v_o, d, 0),\ j = (v_i, v_o-1, d, e) \in \Phi, \\ & \text{and } a = 0 \in A(i); \\ (B-\psi)\mu_d\tau_i(a), & \forall i = (v_i, v_o, d, 0),\ j = (v_i, v_o, d-1, e) \in \Phi, \\ & \text{and } a = 0 \in A(i); \end{cases} \tag{4}$$

Our objective is to minimize a cost function formed by the weighted blocking of real time calls, *i.e.*:

$$C_i(a) = C_{vi}(i,a) + C_{vo}(i,a) \tag{5}$$

where $C_{vi}(i,a)$ and $C_{vo}(i,a)$ are, respectively, the blocking cost of video and voice calls computes as:

$$C_{vi}(i,a) = c_{vi},\ \forall\ e = 1 \text{ and } a = 0 \in A(i), \tag{6}$$

and

$$C_{vo}(i,a) = c_{vo},\ \forall\ e = 2 \text{ and } a = 0 \in A(i), \tag{7}$$

where c_{vi} and c_{vo} are, respectively, the immediate cost incurred whenever an incoming video or voice call ($e = 1, 2$) is blocked. With $\tau_i(a)$, $p_{ij}(a)$ and $C_i(a)$, using the value iteration algorithm and the uniformization method [15] , we can obtain the optimum stationary policy for the system. A stationary policy R, defined by the decision rule $f : \Phi \to A$, prescribes the action $f(i) \in A(i)$ each time the system is observed in the state $i \in \Phi$.

3.1 QoS performance Metrics

Let the state $i \in \Phi$ and the action $a = 1 \in A(i)$. The mean carried real time call (video or voice) traffic is computed as:

$$O_e = \sum_{\forall i \in \Phi, e=1 or 2, a=1 \in A(i)} \tau_i(a)^{-1}\pi_i. \tag{8}$$

where π_i is the steady state probability distribution of the SMDP computed after the optimal policy being found. Giving O_e, we can derive the blocking probability of real time calls as follows:

$$P_{ve} = 1 - \frac{O_e}{\lambda_{ve}}. \tag{9}$$

where ve depends on the value of e. Thus, if $e = 1$, then $ve = vi$; and if $e = 2$, then $ve = vo$. The blocking probability of data call is given by the probability of an incoming data call faces less than the minimum bandwidth; which is:

$$P_{dc} = \sum_{d \geq \lfloor \frac{B-\psi}{B_d} \rfloor} \pi_i \tag{10}$$

The link utilization is given by:

$$U = \frac{B_{vi} \sum_{v_i>0} v_i \pi_i + B_{vo} \sum_{v_o>0} v_o \pi_i + \sum_{d>0} (B-\psi)\pi_i}{B} \tag{11}$$

4. RESULTS

In this section, we present numerical results for the optimal policy. We evaluate the performance of the systems by considering and increasing video call arrival rate and analyzing the impact of the most restricted service classes on the performance of the others. In doing so, we vary λ_{vi} from 1 to 9 calls/hour (0.00027, 0.00055, 0.00083, 0.00111, 0.00138, 0.00166, 0.00194, 0.00222, and 0.0025 call/s) and set the voice call and data call arrival rates in 0.00278 call/s and 0.08 call/s, respectively. The remainder parameters are summarized in Table 1. These values are based in basic parameters found in literature [3, 16] .

Table 1. Used parameters on experiments.

Parameter	**Value**
Downstream data rate (B)	16 Mbps
Bandwidth of video Call (B_{vi})	5 Mbps (MPEG-2)
Bandwidth of voice Call (B_{vo})	64 kbps
Minimum bandwidth of internet call (B_d)	56 kbps
Video call mean duration	5400 seconds
Audio call mean duration	120 seconds
Data call mean duration	3600 seconds
Video blocking cost	10
Audio blocking cost	2

Figure 1 shows the blocking probability for MPEG-2 video traffic for an increasing mean video arrival rate. Because the huge amount of bandwidth required by that service, we can note that its blocking probability quickly increases as λ_{vi} increases. Figure 2 depicts the blocking probability for voice traffic. We can see that for light video traffic, the optimal policy keeps lower blocking probability, but as λ_{vi} increases, it also increases; however, its values are still keep lower. This is also due to the small amount of bandwidth required by that service when comparing with video service's requirement.

Figure 3 shows the blocking probability for data traffic. As expected, the performance of that service also increases as λ_{vi} increases. It is worthwhile to emphasize that since data traffic is served as a best-effort service and it is not take into account in the cost function, its performance may be seriously degrade whether the offered traffic of the real time service being not optimally controlled. For now, a simple way of mitigating the effect of that lack of priority it is to decrease its requirement of bandwidth for each internet section. Besides that problem, this simple solution will also contribute to decrease the mean number of preempted and dropped data call.

In Figure 4, we have the resource utilization of the system. Voice and best effort traffic present an almost constant utilization, 6.5% and 0.13%, respectively. Video traffic as expectet has a high resource utilization.

Figure 5 shows the policy cost for our modeling. Once the video has a higher cost and higher bandwidth requirement, the total cost increases when λ_{vi} increases also.

5. CONCLUSION

In this paper, we have modeled an ADSL2+ system using SMDP and we have defined an optimal transmission strategy for a triple play traffic. We have used three traffic classes multiplexed over a ADSL2+ channel, where each traffic class is characterized by a minimum data rate and blocking cost. For the sake of modeling, we have assumed that call arrivals follows the Poisson processes and the call durations are exponentially distributed, which allows us to model the system as a SMDP. This assumption are justified for traditional voice users and other types of applications [11] . Our model allows an analysis for scenarios where the service provider has to minimize a cost function formed by weighed blocking sum of VoD service and VoIP service when Internet traffic is served by best-effort service of IPV4. We also assume that when an incoming real time call arrives and the Controller decides to accepted it, the bandwidth of the existing Internet call are reduced in order to accommodate such real time call. This may result in a preemption and drop of some data calls. However, we have considered the elastic nature of TCP traffic and ensured the minimum QoS for Internet traffic by setting a minimum bandwidth for that service.

The blocking costs defined in Table 1 and used in the experiments are an initial approach for this study. These values depends of QoS requirements of the incoming services supported by the network and the purpose of DSL link; i.e., if we are design an ADSL2+ link for residential or corporative users.

So far, we have not include in the cost function a way to protect the Internet service against the high offered traffic of real time services. Currently, we are considering to add an objective that try to quell the preemption and drop of data calls. These results will be subject of future works. Future generations of DSL networks as VDSL2 will support 100 Mbps. If we consider that the current model for 16 Mbps yields in a system with: 175098 states and 281001 state-action pairs; we can figure that these networks will pose a big challenge: how to optimally analyze them? Due to the curse of dimensionality, we are also considering using the reinforcement learning solution for SMDP.

Other subject of great impact on the design of DSL networks that we suggest for future works is the decision making about an optimal transmission strategy that chooses between interleaving and Reed-Solomom (RS) code parity. That analysis is extremely important because depending on the decision made, ongoing calls will be more or less protected against noise in the link.

ACKNOWLEDGMENTS

Authors would like to thank the Research and Development Centre, Ericsson Telecomunicacões S.A., Brazil, Federal University of Pará, Department of Electrical and Computing Engineering, Belém, Pará, Brazil. And Ericsson AB, Access Signal Processing Laboratory, Älvsjö, Sweden. Some of the authors want to acknowledge the financial support received from the European Commission IST 6th Framework and from the Swedish Agency for Innovation Systems, VINNOVA, through the IST - MUSE and the Eureka - Celtic BANITS projects respectively, which partially enabled this work.

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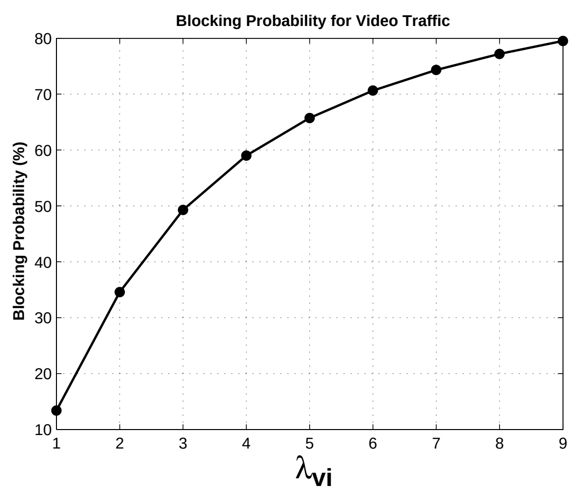

Figure 1. Blocking probability for video traffic against λ_{vi}.

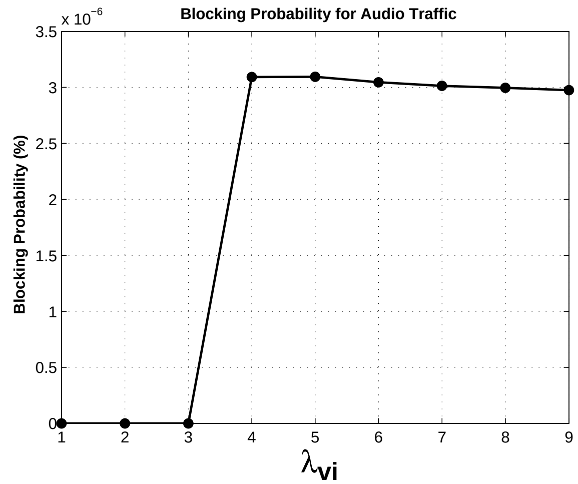

Figure 2. Blocking probability for audio traffic against λ_{vi}.

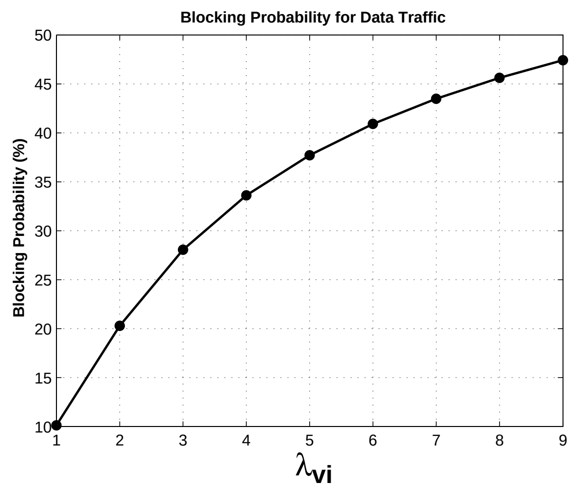

Figure 3. Blocking probability for best effort traffic against λ_{vi}.

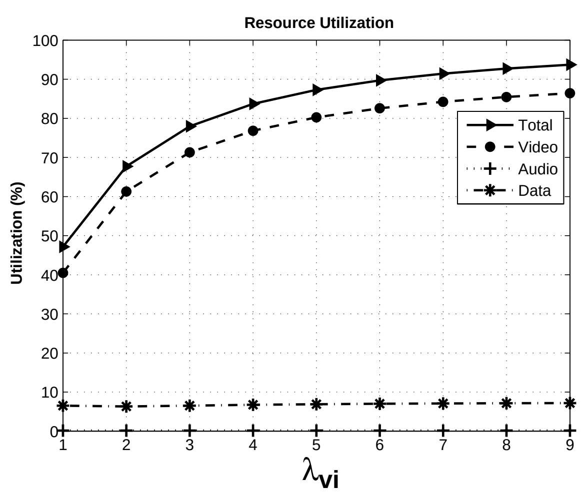

Figure 4. Resource utilization for different classes of traffic against λ_{vi}.

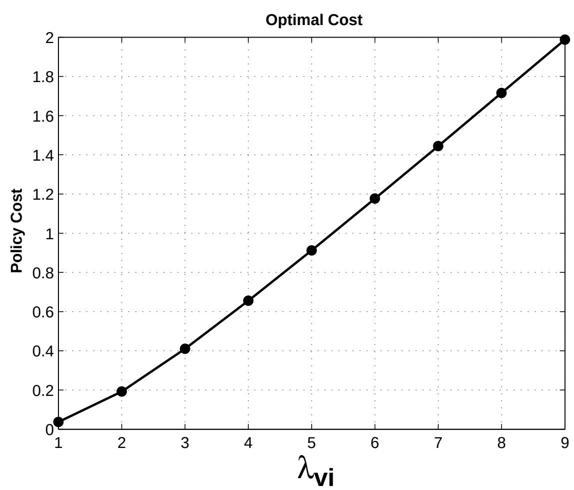

Figure 5. Policy cost against λ_{vi}.