# The Influence of Material Parameters on the Frozen Modes of Magnetic Photonic Crystals

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*Abstract* — **Multilayered structures known as Photonic Crystals are object of many researches lately. Some special Photonic Crystals, containing magnetic layers and misaligned anisotropic dielectric layers, show an asymmetric dispersion relation. These structures are referred as Magnetic Photonic Crystals (MPC). The asymmetric dispersion relation can be associated with the electromagnetic unidirectionality. A unidirectional medium "freezes" the radiation of certain frequency and direction (frozen mode), being perfectly transparent for the wave with the same frequency and the opposite direction. Using the transfer matrix method, we study the influence of various physical and geometrical parameters on the frozen modes of Magnetic Photonic Crystals.**

*Index Terms* — **Optical arrays, optical beams, optical materials, nonreciprocal wave propagation, physical optics.**

## I. Introduction

Photonic crystals are multilayered structures that have been object of many studies in the last years. These systems allow us to control the flux of the light with considerable flexibility. The simplest form of a Photonic Crystal is a periodic array of isotropic dielectric layers with different refraction indexes. This structure displays a Photonic Band Gap, implying that waves of certain frequency don't propagate along the crystal. However, the dispersion relation of this type of Photonic Crystals is perfectly symmetric, namely

$$\omega(\vec{k}) = \omega(-\vec{k}) \,, \qquad (1)$$

where $\omega$ is the wave frequency and $\vec{k}$ is the Bloch wave number.

Besides, the dispersion relation of this type of structure doesn't contain a Stationary Inflection Point (SIP), which is associated to the frozen mode. The frozen mode cannot exist in Photonic Crystals where (1) holds [2]. Therefore, isotropic dielectric structures don't interest to us.

The simplest array that supports spectral asymmetry, with possibility of electromagnetic unidirectionality, is composed of unit cells with three layers: two misaligned anisotropic dielectric layers (A layers) and one magnetic layer (F layer). The spectral asymmetry is represented by the equation below:

$$\omega(\vec{k}) \neq \omega(-\vec{k}) \,. \qquad (2)$$

If the two main requirements below are satisfied, the electromagnetic unidirectionality can be obtained [2]:

(i) The misalignment angle ($\varphi_A$) between the two A layers must be different of 0 and $\pi/2$,

(ii) The F layer must display significant circular birefringence at the frequency range of interest.

The unit cell of this structure is shown in Fig. 1.

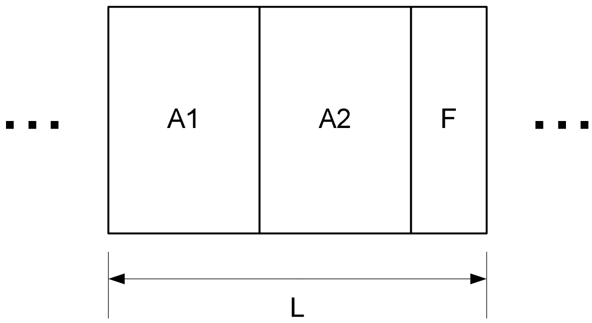


Fig. 1. Representation of the simplest periodic array that shows an asymmetric spectrum. A1 and A2 layers are the dielectric layers, F layer is the magnetic layer and L is the length of the unit cell.

We suggest in this work an idea of frozen modes mapping, i.e., calculation of the dependence of the Stationary Inflection Point ($\omega_0$, $k_0$) on physical and geometrical parameters. This information can be useful for engineering purposes of photonic crystals with frozen modes.

## II. Material Parameters

The A layers and F layers are represented mathematically by material tensors $\hat{\varepsilon}$ (electric permittivity) and $\hat{\mu}$ (magnetic permeability). In this work, for A layers, the two tensors are defined accordingly by two expressions below:

$$\hat{\varepsilon}_A = \varepsilon_0 \begin{bmatrix} \varepsilon_A + \delta_A \cos(2\varphi_A) & \delta_A \sin(2\varphi_A) & 0 \\ \delta_A \sin(2\varphi_A) & \varepsilon_A - \delta_A \cos(2\varphi_A) & 0 \\ 0 & 0 & \varepsilon_{ZZ} \end{bmatrix}, \qquad (3a)$$

$$\hat{\mu}_A = \mu_0 \mu_R \hat{I} \,, \qquad (3b)$$

where $\varepsilon_A$ is the electric permittivity, $\delta_A$ is the magnitude of in-plane anisotropy, $\varphi_A$ is the misalignment angle between A layers and $\hat{I}$ is the identity matrix.

For F layers we have the following tensors:

$$\hat{\varepsilon}_F = \varepsilon_0 \varepsilon_R \hat{I}\,, \qquad (4a)$$

$$\hat{\mu}_F = \mu_0 \begin{bmatrix} 1+\chi_{XX} & \chi_{XY} & 0 \\ \chi_{YX} & 1+\chi_{YY} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (4b)$$

where

$$\chi_{XX} = \chi_{YY} = \chi'_{XX} + j\chi''_{XX}\,, \qquad (5a)$$

$$\chi'_{XX} = \frac{\omega_M \omega_H (\omega_H^2 - \omega^2) + \omega_M \omega_H \omega^2 \alpha^2}{[\omega_H^2 - \omega^2(1+\alpha^2)]^2 + 4\omega_H^2 \omega^2 \alpha^2}\,, \qquad (5b)$$

$$\chi''_{XX} = \frac{-\omega_M \omega \alpha[\omega_H^2 + \omega^2(1+\alpha^2)]}{[\omega_H^2 - \omega^2(1+\alpha^2)]^2 + 4\omega_H^2 \omega^2 \alpha^2}\,, \qquad (5c)$$

$$\chi_{XY} = -\chi_{YX} = j(\chi'_{XY} + j\chi''_{XY})\,, \qquad (5d)$$

$$\chi'_{XY} = \frac{-\omega_M \omega[\omega_H^2 - \omega^2(1+\alpha^2)]}{[\omega_H^2 - \omega^2(1+\alpha^2)]^2 + 4\omega_H^2 \omega^2 \alpha^2}\,, \qquad (5e)$$

$$\chi''_{XY} = \frac{-2\omega_M \omega_H \omega^2 \alpha}{[\omega_H^2 - \omega^2(1+\alpha^2)]^2 + 4\omega_H^2 \omega^2 \alpha^2}\,. \qquad (5f)$$

In the above, $\omega$ is the wave frequency, $\alpha$ is the damping constant, $\omega_H = \gamma H_0$, $\omega_M = \gamma 4\pi M_S$, where $\gamma$ is the gyromagnetic ratio, $H_0$ is the DC biasing magnetic field amplitude and $M_S$ is the DC saturation magnetization [3].

Furthermore, another important parameter related to the periodic array is the ratio $\rho$, which relates the thickness of A layers and F layers. This parameter is defined by the equation below:

$$\rho = L_F / L_A\,, \qquad (6)$$

where $L_F$ is the thickness of the F layers and $L_A$ is the thickness of the A layers.

## III. The Frozen Mode

The effect of electromagnetic unidirectionality in a Magnetic Photonic Crystal is a function of various material parameters of the periodic array. Photonic Crystals that show the electromagnetic unidirectionality freeze the waves of a given frequency and are perfectly transparent for waves of same frequency and opposite direction.

At the viewpoint of the dispersion relation (band diagram), the frozen mode is associated with a Stationary Inflection Point (SIP), in other words, to a point ($\omega_0$, $k_0$) of the band diagram. Only one frozen mode can appear for a given frequency $\omega_0$. The SIP has the following property:

$$\left.\frac{\partial \omega}{\partial k}\right|_{k=k_0} = 0\,;\ \left.\frac{\partial^2 \omega}{\partial k^2}\right|_{k=k_0} = 0\,,\ \left.\frac{\partial^3 \omega}{\partial k^3}\right|_{k=k_0} \neq 0\,. \qquad (7)$$

In Fig. 2 we have an example of a band diagram containing a SIP:

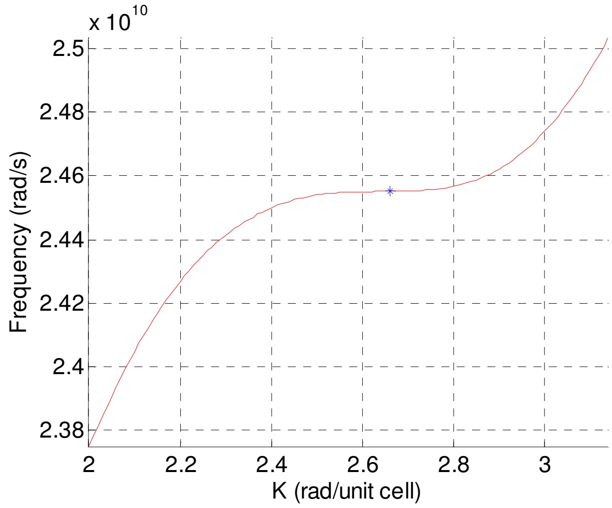


Fig. 2. The vicinity of a SIP. In this figure, SIP is represented by a blue point.

Considering that the two main requirements for the electromagnetic unidirectionality written in section I are satisfied, we can always obtain a frozen mode at a given frequency $\omega_0$ by adjusting at least two material parameters of the periodic stack, such as:

i) The magnetic permeability and/or electric permittivity of layers,
ii) The misalignment angle $\varphi_A = \varphi_{A1} - \varphi_{A2}$,
iii) The ratio $\rho = L_F/L_A$.

In this work we perform a series of simulations that proves the affirmations above.

## IV. Simulations

### *A. Mathematical Model*

In order to study the effects of changes in the physical or geometrical parameters on the band diagram of a Magnetic Photonic Crystal, we need a mathematical model for this structure. We use in this paper the method of transfer matrix, which is simple to make a computational program and it produces accurate results.

Basically, this method consists in obtain a transfer matrix for each layer of the unit cell. The Maxwell equations, (4a), (4b), (5a) and (5b) are used to do this. More information about this can be found in [1]. With the transfer matrixes of all layers (A layers and F layers), we can determine the transfer matrix of a unit cell. This transfer matrix corresponds to the product between the transfer matrixes of all layers that compose the unit cell, namely

$$\hat{T}_S = \prod_M \hat{T}_M \,, \qquad (8)$$

where $\hat{T}_S$ is the transfer matrix of a single unit cell and $\hat{T}_M$ is the transfer matrix of a single layer.

The characteristic polynomial of (8) is used to compute the band diagram $\omega(\vec{k})$ of the Photonic Crystal. It has the form

$$F(z) = Z^4 + P_3 Z^3 + P_2 Z^2 + P_1 Z + 1 = 0 \,, \qquad (9)$$

where

$$Z = \cos(k) + j\sin(k) \,. \qquad (10)$$

The coefficients of (9) are function of the frequency $\omega$. Using (9) and (10) we compute the band diagram $\omega(\vec{k})$ of a Photonic Cristal.

*B. MPC Parameters*

As we have seen, the band diagram of a Photonic Crystal is a function of the material tensors defined in (4a), (4b), (5a) and (5b). Besides, it is a function of ratio (6) and the misalignment angle $\varphi_A$.

We will consider a basic set of material parameters and, from this original MPC, we will obtain new band diagrams by adjusting two parameters of the periodic stack. The original parameters are as follows [3]:

i) Geometrical parameters: $L_{A1} = L_{A2} = 5\text{mm}$, $L_F = 1\text{mm}$ and $\rho = 0.1$,

ii) A layers: $\varepsilon_{A1} = \varepsilon_{A2} = 7$; $\delta_{A1} = \delta_{A2} = 6$; $\varphi_{A1} = 0$, $\varphi_{A2} = 36.0963°$; $\varepsilon_{ZZ,A1} = \varepsilon_{ZZ,A2} = 1$; $\mu_{R,A1} = \mu_{R,A2} = 1$,

iii) F layers: $\varepsilon_R = 5$; $\alpha = 0$; $\omega_H = 36.503\text{x}10^9 \,\text{rad/s}$; $\omega_M = 73.006\text{x}10^9 \,\text{rad/s}$.

*C. Band Diagrams*

Considering the specifications of the Magnetic Photonic Crystal whose band diagram is presented on Fig. 2 and the material parameters are depicted in the previous subsection, we will modify this MPC by adjusting just two parameters of the periodic array and obtain new band diagrams. All the band diagrams obtained from the original one on Fig. 2 display the SIP, associated with the frozen mode. Changing one of the parameters, we calculate another one giving a SIP, but in general, the new SIP will be different. We can obtain a frozen mode by adjusting at least two physical or geometrical parameters of the periodic array.

i) Adjusting $\varepsilon_A$ and $\omega_H$.

In this case we shown that, for a given value of electric permittivity $\varepsilon_A$ (related to the A layers, $\varepsilon_A = \varepsilon_{A1} = \varepsilon_{A2}$), one can obtain a band diagram with a frozen mode by adjusting the value $\omega_H$ (related to the F layer). In our simulations we have obtained some sets for $\varepsilon_A$ and $\omega_H$ for which the band diagram of the structures contains a frozen mode (SIP). We can view this in the table below:

TABLE I

SUMMARY OF VALUES FOR $\varepsilon_A$ AND $\omega_H$ THAT LEADS TO A SIP

| Set | $\varepsilon_A$ | $\omega_H$ (rad/s) | $\omega_0$ (rad/s) | $k_0$ |
|---|---|---|---|---|
| 1 | 5.0 | 34.000x10$^9$ | 2.6282x10$^{10}$ | 2.49 |
| 2 | 5.5 | 34.700x10$^9$ | 2.5843x10$^{10}$ | 2.54 |
| 3 | 6.0 | 35.200x10$^9$ | 2.5386x10$^{10}$ | 2.58 |
| 4 | 6.5 | 35.900x10$^9$ | 2.4968x10$^{10}$ | 2.63 |
| 5 | 7.0 | 36.503x10$^9$ | 2.4551x10$^{10}$ | 2.66 |
| 6 | 7.5 | 37.300x10$^9$ | 2.4166x10$^{10}$ | 2.71 |
| 7 | 8.0 | 38.000x10$^9$ | 2.3784x10$^{10}$ | 2.74 |
| 8 | 8.5 | 35.200x10$^9$ | 2.3436x10$^{10}$ | 2.79 |
| 9 | 9.0 | 35.200x10$^9$ | 2.3085x10$^{10}$ | 2.80 |
| 10 | 9.5 | 40.800x10$^9$ | 2.2756x10$^{10}$ | 2.84 |

We can represent these sets in the figure below:

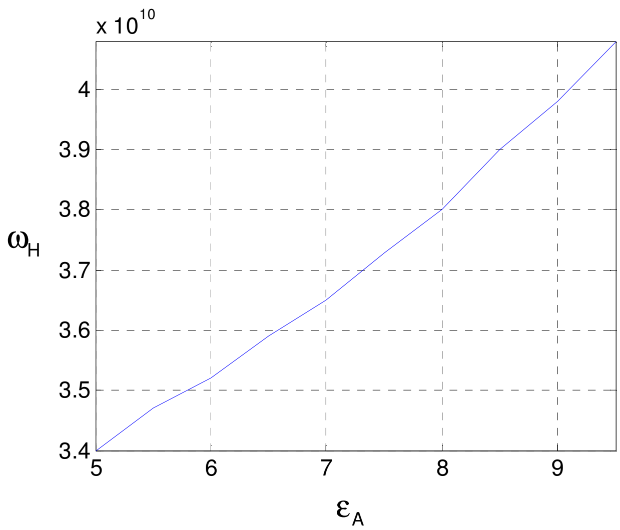


Fig. 3. Values of $\varepsilon_A$ and $\omega_H$ for which the band diagram of the structure shows a frozen mode.

ii) Adjusting $\varphi_A$ and $\omega_H$.

Now considering adjusts in the value of the misalignment angle $\varphi_A$ (related to the A layers) and in the value $\omega_H$ (related to the F layer), we have determined the following table:

TABLE II

SUMMARY OF VALUES FOR $\varphi_A$ AND $\omega_H$ THAT LEADS TO A SIP

| Set | $\varphi_A$ | $\omega_H$ (rad/s) | $\omega_0$ (rad/s) | $k_0$ |
|---|---|---|---|---|
| 1 | 20.0000° | 30.200x$10^9$ | 2.4640x$10^{10}$ | 2.59 |
| 2 | 25.0000° | 31.700x$10^9$ | 2.4571x$10^{10}$ | 2.58 |
| 3 | 30.0000° | 33.600x$10^9$ | 2.4544x$10^{10}$ | 2.61 |
| 4 | 35.0000° | 36.000x$10^9$ | 2.4553x$10^{10}$ | 2.66 |
| 5 | 36.0963° | 36.503x$10^9$ | 2.4551x$10^{10}$ | 2.66 |
| 6 | 40.0000° | 38.900x$10^9$ | 2.4586x$10^{10}$ | 2.70 |
| 7 | 45.0000° | 42.500x$10^9$ | 2.4641x$10^{10}$ | 2.71 |
| 8 | 50.0000° | 47.600x$10^9$ | 2.4750x$10^{10}$ | 2.78 |
| 9 | 55.0000° | 54.500x$10^9$ | 2.4884x$10^{10}$ | 2.82 |
| 10 | 60.0000° | 64.300x$10^9$ | 2.5046x$10^{10}$ | 2.89 |

and the following graph:

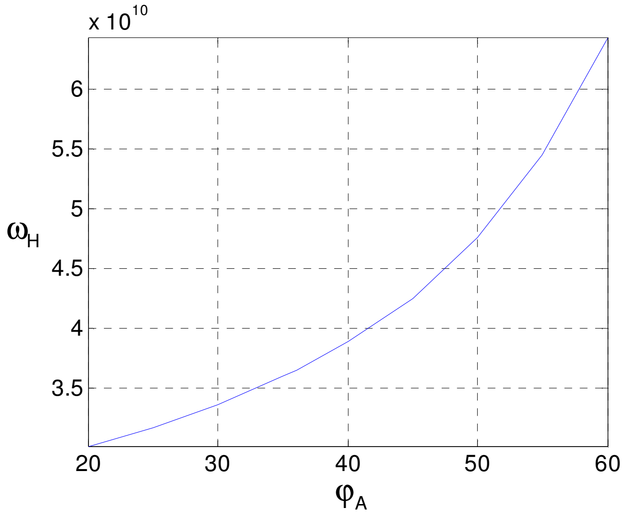


Fig. 4. Values of $\varphi_A$ and $\omega_H$ for which the band diagram of the structure shows a frozen mode.

iii) Adjusting $\varepsilon_A$ and $\rho$.

At last, we have considered a variation of the electric permittivity $\varepsilon_A$ (related to the A layers) and the value of the ratio $\rho$ (related to the thicknesses of A and F layers). Table II represents the sets of values for $\varepsilon_A$ and $\rho$ that leads to a frozen mode in the band diagram of the structure.

TABLE III

SUMMARY OF VALUES FOR $\varepsilon_A$ and $\rho$ THAT LEADS TO A SIP

| Set | $\varepsilon_A$ | $\rho$ | $\omega_0$ (rad/s) | $k_0$ |
|---|---|---|---|---|
| 1 | 5.0 | 0.140 | 2.5800x$10^{10}$ | 2.54 |
| 2 | 5.5 | 0.125 | 2.5542x$10^{10}$ | 2.59 |
| 3 | 6.0 | 0.115 | 2.5217x$10^{10}$ | 2.59 |
| 4 | 6.5 | 0.105 | 2.4923x$10^{10}$ | 2.66 |
| 5 | 7.0 | 0.100 | 2.4551x$10^{10}$ | 2.66 |
| 6 | 7.5 | 0.093 | 2.4244x$10^{10}$ | 2.71 |
| 7 | 8.0 | 0.088 | 2.3917x$10^{10}$ | 2.74 |
| 8 | 8.5 | 0.084 | 2.3587x$10^{10}$ | 2.76 |
| 9 | 9.0 | 0.080 | 2.3276x$10^{10}$ | 2.78 |
| 10 | 9.5 | 0.078 | 2.2935x$10^{10}$ | 2.76 |

These sets are represented in the next figure:

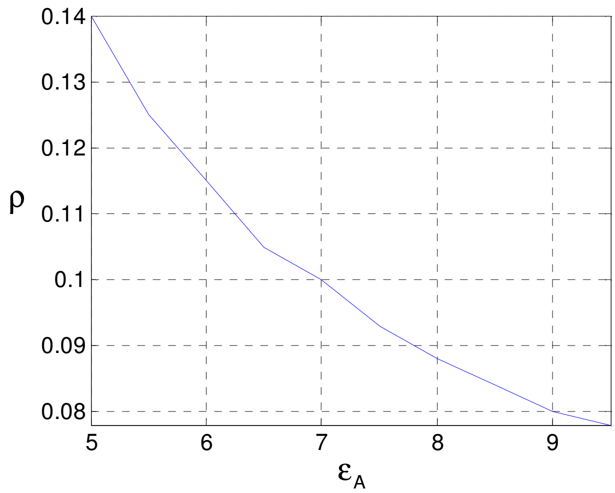


Fig. 5. Values of $\varepsilon_A$ and $\rho$ for which the band diagram of the structure shows a frozen mode.

## V. CONCLUSION

We have performed a series of simulations, modifying various parameters of the original MPC and, for the sets depicted in the tables I, II and III, we always have obtained a frozen mode in the band diagram.

We have shown that changing the material and geometric parameters, one can change the position of the Stationary Inflection Point on the $\omega - k$ diagram. Using practical limits of the physical parameters, one can define the limits of the $(\omega_0, k_0)$ position on the band diagram. These results can be useful for the purposes of engineering of photonic crystals with frozen modes.

## ACKNOWLEDGEMENT

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