THEORETICAL ANALYSIS OF A MODIFIED KOCH MONOPOLE WITH REDUCED DIMENSIONS

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ABSTRACT: *Fractal antennas are characterized by their special geometric characteristics which allow a reduction of the antenna dimensions. In this work, we analyze the fractal parameters and the radiative properties of modified Koch monopoles. The geometries of the antennas are obtained by an ad hoc Iterative Function System algorithm for fractal curves generation. Using the Method of Moments for numerical calculations we analyze the influence of the antenna geometry on the resonant frequency, current, efficiency, input impedance and radiation pattern.*

Key words: *Monopole antennas, fractal antennas, modified Koch monopole.*

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I. INTRODUCTION

Miniaturization of antennas is one of the trends in modern communications systems [1]. One of the techniques used to decrease the antenna's dimensions is application of fractal geometries [2]. The Koch monopoles are fractal linear antennas which have the geometry of the Koch curves. These antennas can be for example, vertical arrangements mounted above a ground plane. Figure 1 shows four iterations of this fractal. The radiation properties of these monopoles are discussed in [3-6]. The reduction of the resonant frequency as a function of the number of iterations converges asymptotically to the limit around 44% [3]. For the conventional Koch curve, the angle α (which is called the indentation angle) in Figure 1 is 60º and the parameters s_i are equal, i.e. $s_1=s_2=s_3=s_4$.

In this paper, we investigate another variant of the Koch monopoles with dimensions smaller then the conventional Koch monopoles. In our modified Koch fractals, we fix the values $s_1=s_4$ equal to one third of the total height L (Figure 1) and change the angle α scaling the values of $s_2=s_3$ in order to preserve the height L constant for all iterations. This leads to a change of the geometry and of the fractal dimension. An ad hoc Iterative Function System (IFS) algorithm is used to model the antenna's geometry. For numerical calculations of the input impedance, resonant frequency, efficiency, current along the conductor and radiation pattern we use the Method of Moments (MoM) [7].

II. FRACTAL DESCRIPTION

There is a method to produce the fractals called initiator-generator construction [2]. In this method, one begins with a specified initiator, and a generator is applied repeatedly in a lower scale to form the fractals. In Figure 1, for example, K_0 is the initiator with the length L, and K_1 is the generator. Below, we describe the ad hoc IFS algorithm, which is used to obtain the modified Koch fractals and show the geometrical properties of these fractals.

A. IFS Algorithm. The initiator with α=0° corresponds to the straight monopole. In order to construct the generator, we apply the four affinity transformations W_1, W_2, W_3 and W_4 at the points that define the initiator's curve and join the four obtained segments. These four transformations can be applied successively to construct the monopoles K_1, K_2, …, K_n. The procedure can be represented symbolically by

$$K_{n+1} = W(K_n) = \bigcup_{p=1}^{4} W_p(K_n) = W_1(K_n) \cup W_2(K_n) \cup W_3(K_n) \cup W_4(K_n), \tag{1}$$

where n is the nth fractal iteration. Considering the plane xz (Figure 1) and supposing that K_0 is on the axis $+z$ with one of their extremities at the origin, the transformations are defined by the following expressions:

$$W_1\begin{pmatrix} z \\ x \end{pmatrix} = \begin{bmatrix} L/3 & 0 \\ 0 & L/3 \end{bmatrix}\begin{pmatrix} z \\ x \end{pmatrix}, \tag{2}$$

$$W_2\begin{pmatrix} z \\ x \end{pmatrix} = \begin{bmatrix} (L/e_1)\cos\alpha & -(L/e_1)\operatorname{sen}\alpha \\ (L/e_1)\operatorname{sen}\alpha & (L/e_1)\cos\alpha \end{bmatrix}\begin{pmatrix} z \\ x \end{pmatrix} + \begin{pmatrix} L/3 \\ 0 \end{pmatrix}, \tag{3}$$

$$W_3\begin{pmatrix} z \\ x \end{pmatrix} = \begin{bmatrix} (L/e_1)\cos\alpha & (L/e_1)\operatorname{sen}\alpha \\ -(L/e_1)\operatorname{sen}\alpha & (L/e_1)\cos\alpha \end{bmatrix}\begin{pmatrix} z \\ x \end{pmatrix} + \begin{pmatrix} L/2 \\ (L/6)\tan\alpha \end{pmatrix}, \tag{4}$$

$$W_4\begin{pmatrix} z \\ x \end{pmatrix} = \begin{bmatrix} L/3 & 0 \\ 0 & L/3 \end{bmatrix}\begin{pmatrix} z \\ x \end{pmatrix} + \begin{pmatrix} 2L/3 \\ 0 \end{pmatrix}, \tag{5}$$

where e_1=6cosα. These formulas are a generalization of the known ones for the conventional Koch fractal. Thus, the conventional Koch monopole can be considered as a particular case of our monopoles with α=60°. Figure 2 shows four iterations of the modified Koch fractals for α=40° and α=70°.

B. Fractal Dimension. The fractal dimension D is a number, which characterizes fractal structures. This parameter can be understood as a measurement of the space filling ability by a fractal form. There are different definitions of D. One of them, which we use here, is the Hausdorff-Besicovich dimension (or self-similarity dimension) [2]. In this definition, the dimension D is the solution of the equation

$$k_1\left(\frac{1}{h_1}\right)^D + k_2\left(\frac{1}{h_2}\right)^D + \ldots + k_m\left(\frac{1}{h_m}\right)^D = 1, \tag{6}$$

where k_m is the number of the copies of the initiator scaled by h_m and m the number of different scale that the fractal possesses. For the fractal described by the relations (1)-(5), we have m=2 and s_1=L/3, s_2=L/e_1, s_3=L/e_1 and s_4=L/3, therefore, k_1=k_2=2, h_1=3 and h_2=6cosα. Substituting these parameters in (6), we obtain

$$\left(\frac{1}{3}\right)^D + \left(\frac{1}{6\cos\alpha}\right)^D = \frac{1}{2}. \tag{7}$$

The solution of this transcendental equation gives the value of D for a given α.

The total length of the wire conductor l_n is an important parameter in the antenna design. Usually, the longer conductor of the antenna, the less resonant frequency of the antenna can be achieved. It can be shown that the length l_n of our modified Koch monopole for the nth iteration can be calculated using the following equation:

$$l_n = 2^n \left[\frac{1}{3} + \frac{1}{6\cos\alpha} \right]^n L. \tag{8}$$

Table I shows the calculated values of l_n and D obtained in four iterations of this fractal for different angles α and L=1m.

TABLE 1 Variation of ln and *D* as a Function of n and α

α	D	Length l_n (normalized with L=1m) of the fractal			
		n=1	n=2	n=3	n=4
10°	1.0038	1.0051	1.0103	1.0155	1.0207
25°	1.0258	1.0345	1.0701	1.1070	1.1451
40°	1.0766	1.1018	1.2140	1.3376	1.4737
55°	1.1905	1.2478	1.5570	1.9429	2.4244
60°	1.2618	1.3333	1.7778	2.3704	3.1605
70°	1.5739	1.6413	2.6938	4.4212	7.2564

III. NUMERICAL RESULTS

Using the IFS algorithm described above we developed a MoM code. This code is based on the theory of [7]. In this code, we use the pulse and Dirac's delta functions for basis and test functions respectively. The parameters of the monopoles are as follows: the height L=6cm, the conductor diameter d=0.1mm. The pre-fractals for iterations K_0, K_1, K_2, K_3 and K_4 are analyzed for different angles α. The numbers of discrete segments in each iteration K_0, K_1, K_2, K_3 and K_4 of our MoM model are 31, 36, 80, 192 and 256, respectively.

A. Resonant Frequency, Radiation Resistance, and Efficiency. In order to verify the developed algorithm, we compare our calculations for the conventional Koch monopole (α=60°) with numerical results obtained in [4]. The maximum difference between our results and those in [4] for K_0 to K_4 for the first resonant frequency is 2.6% and for the radiation resistance 1.5%. Notice that the radiation resistance considered here is the real part of the input impedance Z_{in} of the lossless antenna in the first resonant frequency. In this resonance, the imaginary part of the Z_{in} is null, i.e. X_{in}=0.

In the Figures 3a and 3b, we give the normalized lengths of the antenna L/λ (λ is the wavelength) versus iteration number n with α as a parameter for the first and the second resonance, respectively. Figure 4a exhibits the variation of the radiation resistance R_r and Figure 4b shows the efficiency of these antennas at the first resonance. We can see from these graphics that when the angle α or, equivalently, the dimension D is increased (Table I), the resonant frequencies become smaller. We also observe that each curve in Figure 3a tends asymptotically to a determined limit. For the curve with α=70°, we obtain for K_4 the reduction in the fundamental resonance frequency of approximately 68% in comparison with the conventional monopole of the same height L. The radiation resistance and efficiency of these monopoles are reduced approximately to 4 Ohms and 50%, respectively (Figures 4a and 4b).

B. Input Impedance. The input impedance $Z_{in}=R_{in}+jX_{in}$ of the first four iterations for the modified Koch monopole with α=40° and α=70° are shown in Figures 5 and 6, respectively. One can see that the frequency dependence of Z_{in} of the monopoles with α=40° (Figure 5) is relatively independent of n. In contrast to this, the frequency dependence of the input impedance of the monopoles with α=70° has very large dependence of n. In general, the variation of Z_{in} in respect the frequency becomes greater with increasing α or n.

The presented results show that the reduction of the resonant frequency is accompanied by a reduction of the bandwidths of the antennas. Notice that it is a common feature of fractal antennas.

C. Current Distribution. Figures 7a and 7b show the current distribution for the Koch monopoles with α=70°. In these figures, the normalized current magnitudes I_n along the conductor for the first (Figure 7a) and second (Figure 7b) resonance are plotted. The horizontal axis in these figures is the normalized length l/L along the conductor and l/L=0 is on the top of the antenna (superior extremity). The resonant values of the L/λ used for calculations are given in Figure 3.

From Figure 7, we can observe also that the current distributions at the first and second resonance are similar to those of the conventional monopole (it corresponds to K_0 in Figure 7) with sinusoidal distribution.

D. Radiation Patterns. The radiation patterns for the planes *xz* and *yz* (see the orientation of the coordinate system in Figure 1) of the analyzed antennas for the angles α=40° and α=70° and fourth iteration (K_4) are shown in Figure 8. These figures present the diagrams for the second resonance. For the first resonance, these Koch monopoles possess diagrams similar to those of the monopole K_0, therefore these diagrams are not shown here. On the diagrams of Figure 8, the co- and cross- polarizations are relative to the electric field components in the far zone E_θ and E_ϕ, respectively.

The graphics show that the antenna with α=70° has a considerable radiation in the vertical z-direction. This is due to the fact that some sections of the antenna are oriented almost parallel to the horizontal plane. In spite of the geometrical symmetry of our fractals with respect to their mid points, the horizontal currents with opposite directions in the symmetric sections have different values. Therefore, the radiations of these sections do not compensate each other.

The cross-polarization for the antenna with α=70° is larger then that for the antenna with α=40°. The co-polarization radiation patterns in the plane yz for all the monopoles for any frequency have the null value in the direction z. This is because the monopole fractals lie in the plane xz producing the electric field components in the far zone only in this plane. Our simulations show also that all the analyzed antennas have practically isotropic radiation patterns in the horizontal plane xy.

IV. CONCLUSIONS

A version of the Koch antennas has been considered in this work theoretically. The modified Koch monopoles were constructed by changing the angle α of the generator and preserving the values of s_1=s_4 equal to one third of the total length L (Figure 1) and scaling the values of s_2=s_3. It was shown that the dimensions of the antennas can be reduced by choosing the angle α>60^o (α=60^o for the conventional Koch fractal). For the angle α=70^o, the reduction of the first resonant frequency is about 68% in comparison with the straight monopole (for the conventional Koch monopole this value is 44%). The modified Koch antennas possess lower impedance bandwidth, lower radiation resistance and lower efficiency as compared with the conventional Koch monopoles. As to the radiation pattern, the modification of the Koch antennas does not lead to a significant change at the first resonance. Some changes of the radiation patterns have been observed at the second resonance regime of the modified antennas.

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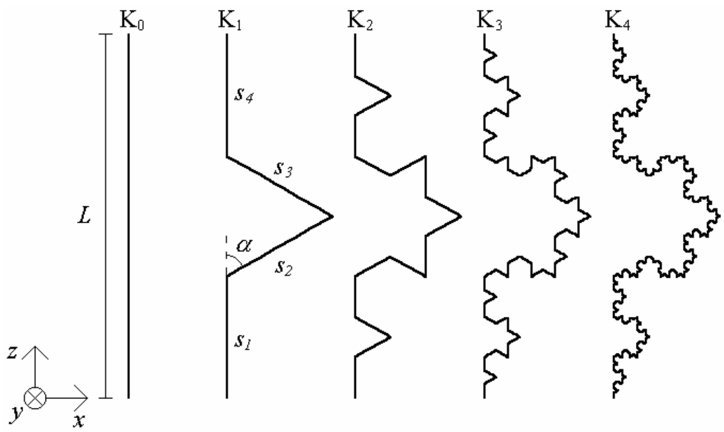


Figure 1 Curves correspondent to the 4 first iterations of the Koch fractal. The monopoles K_0 and K_1 are initiator and generator, respectively.

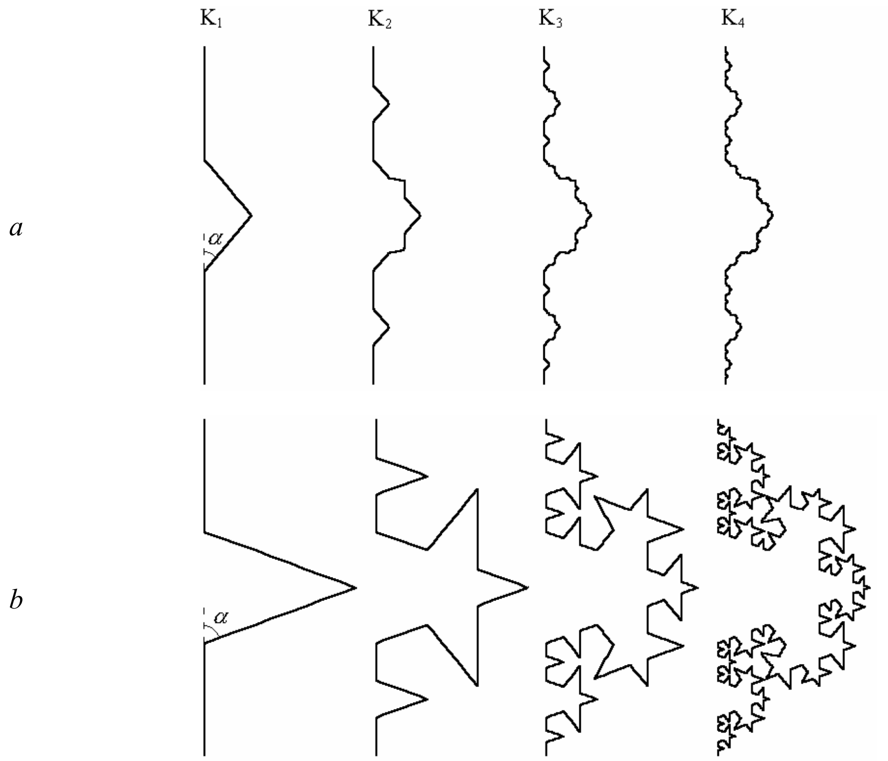
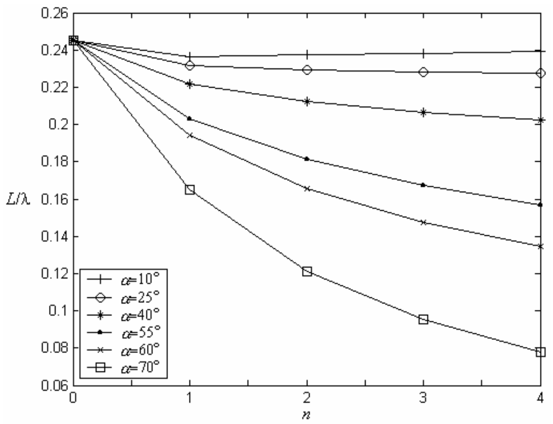


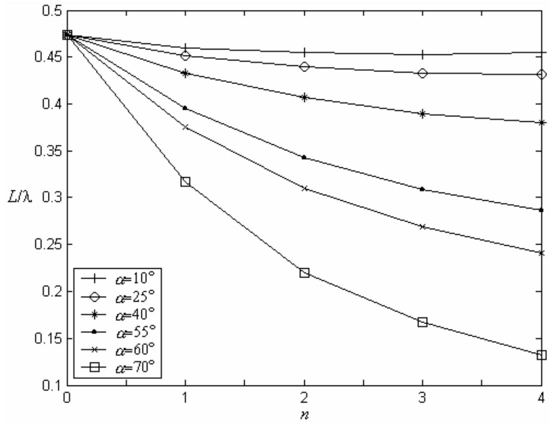
Figure 2 Modified Koch monopoles.

a Monopoles with α=40^0

b Monopoles with α=70^0



a

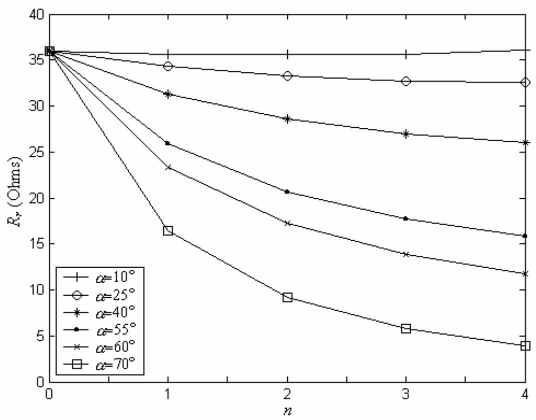


b

Figure 3 Normalized length L/λ of Koch monopoles as a function of α and n.

a First resonance

b Second resonance



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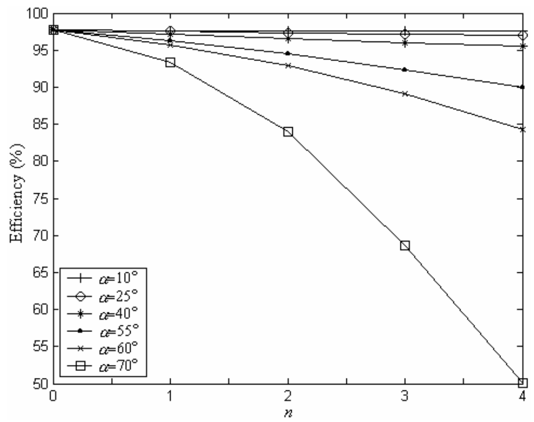


b

Figure 4 Radiation resistance R_r and efficiency of Koch monopoles as a function of α and n. These parameters were calculated at the first resonance.

a Radiation resistance

b Efficiency

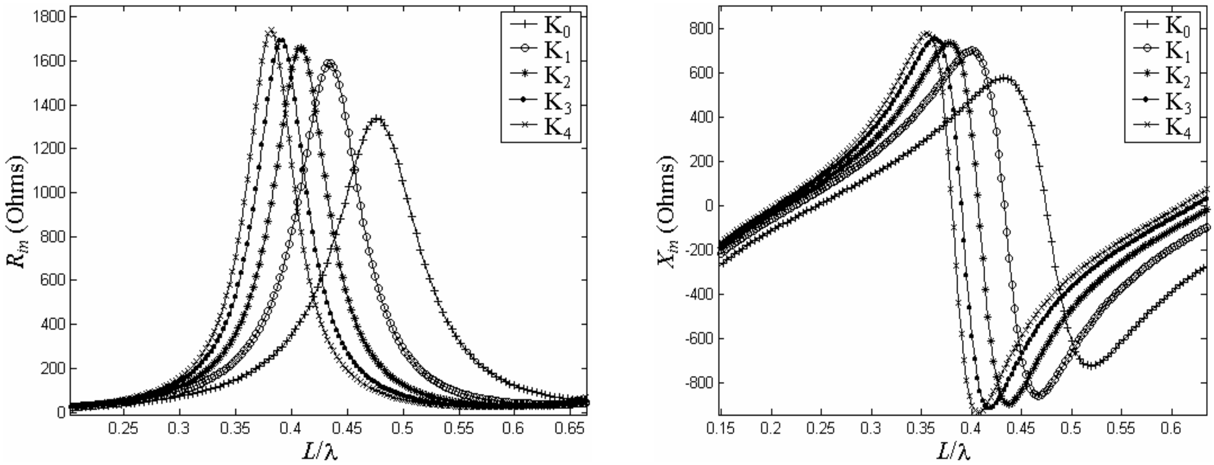
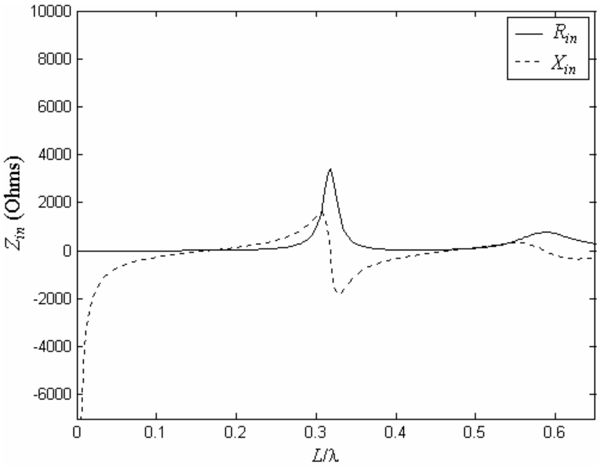
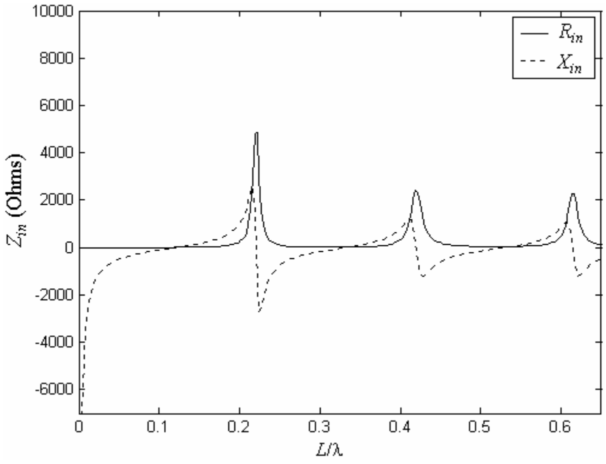


Figure 5 Input impedance for the Koch monopoles with α =40°.



a



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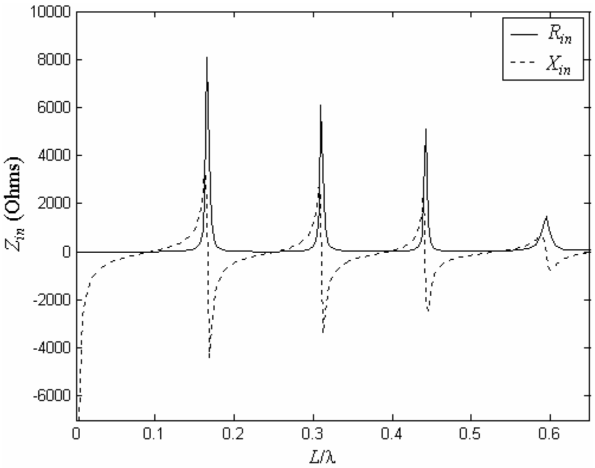
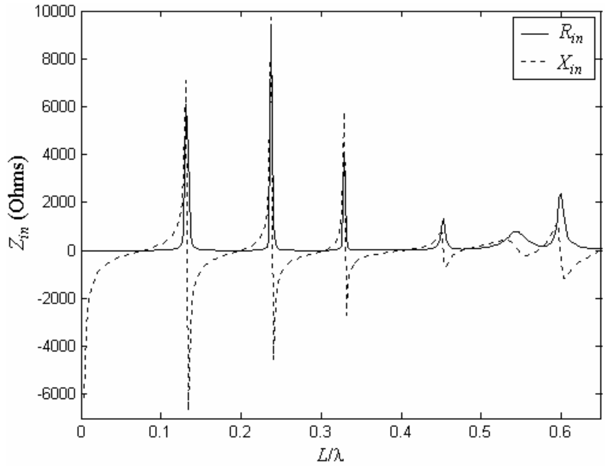


c



d

Figure 6 Input impedances for the Koch monopoles with α=70°.

a K_1

b K_2

c K_3

d K_4

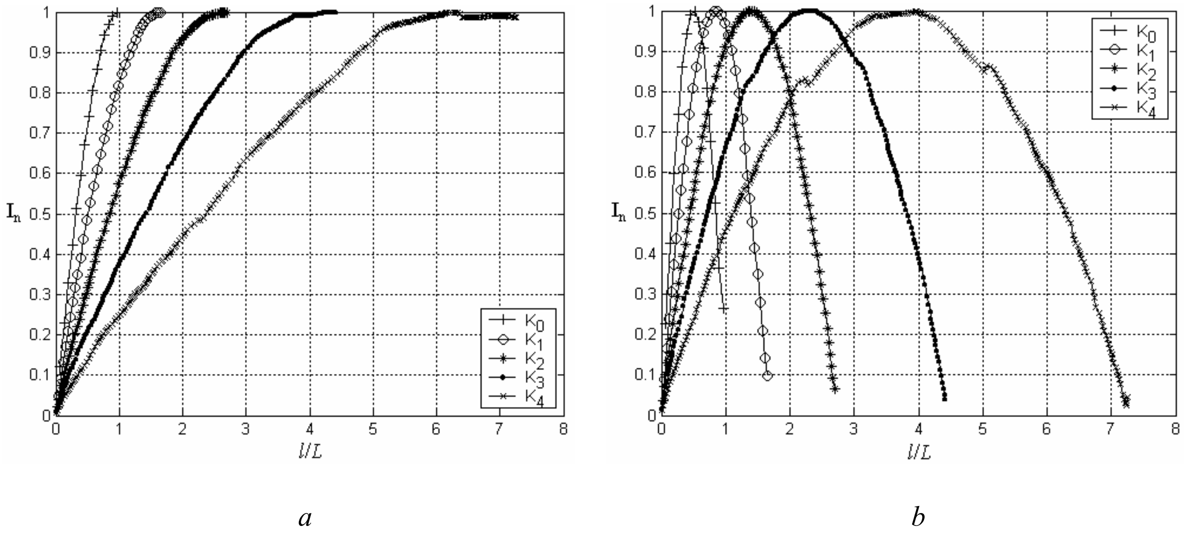


Figure 7 Current distribution for the Koch monopoles with α=70°.

a First resonance

b Second resonance

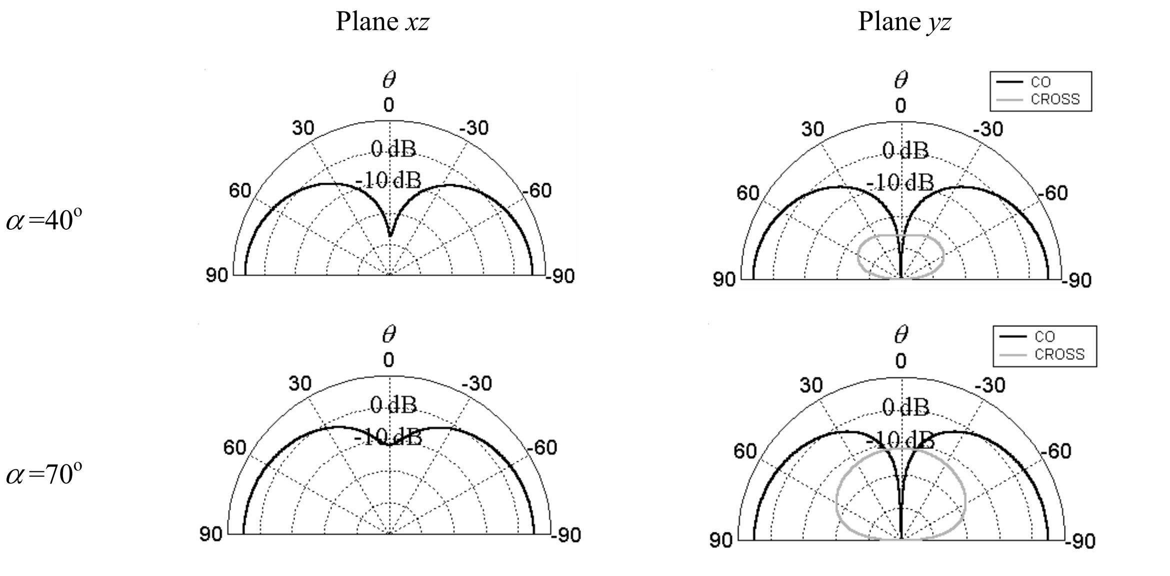


Figure 8 Radiation patterns for the Koch monopoles with α=40° and α=70° at the second resonance and fourth iteration (K_4).