

# A new acceleration technique for the design of fibre gratings

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**Abstract:** In this paper we propose a novel acceleration technique for the design of fibre gratings based on Genetic Algorithm (GA). It is shown that with an appropriate reformulation of the wavelength sampling scheme it is possible to design high quality optical filters with low computational effort. Our results will show that the proposed technique can reduce significantly the GA's processing time.

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## 1. Introduction

Fibre gratings are one of most important developments in the field of optical fibre technology. Their invention was nominated one of the milestone events in the history of optical communications. They are very simple, low cost, distributed reflectors whose reflection and transmission

spectra are wavelength-dependent and can be accurately adjusted by proper design. Gratings are written in fibres making use of the *photosensitivity* of certain types of materials.

Fibre gratings are classified as either short-period (also called Bragg gratings) or long-period gratings. Long-period gratings have periods that are much higher than the wavelength, ranging from a few hundred  $\mu m$  to few millimetres. Short-period gratings have periods that are comparable to the wavelength, typically around  $0.5 \mu m$  [1, 2, 3].

The synthesis methods for fibre gratings can be classified into two categories: The inverse scattering methods [4]; and the optimisation methods [5]. The inverse scattering methods directly compute the required grating profile from the grating reflection or transmission spectra by solving the mathematical inverse problem while the optimisation method directly minimise the difference between the synthesised and targeted spectra. The optimisation can be performed by several optimisation algorithms such as Simulated Annealing [6], Tabu Search [7], Genetic Algorithm (GA) [5, 8] and recently by Particle Swarm Optimisation [9].

In this paper we propose a novel acceleration technique to compute the objective function in GA algorithm based on the stochastic variation of the wavelength samples. The technique will be applied to the problem of fibre grating synthesis. The results will show that our technique can improve GA performance. The outline of this paper is as follows: Section 2 the theoretical approach and the synthesis algorithm are presented, Section 3 contains a short description of Genetic Algorithms and the proposed acceleration technique. Section 4 contains numerical results and comparisons, and finally the main conclusions are drawn in Section 5.

## 2. Theory

In our work the well-known transfer matrix method is applied to solve the couple mode equations and to obtain the spectral response of the fibre grating. In this approach, the grating is divided into uniform sections, each section is represented by a  $2 \times 2$  matrix. By multiplying these matrices, a global matrix that describes the whole grating is obtained. We will describe the approach developed by Yamada et al. [10].

We assume that the refractive index, inside a  $i$ th uniform section from a Bragg grating, can be described by

$$n(z) = n_{eff} + \Delta n_i \cos(2\pi z / \Lambda_i). \quad (1)$$

Where  $n_{eff}$  is the effective index of fibre core,  $\Delta n_i$  is the refractive index amplitude modulation, and  $\Lambda_i$  is the section grating period.

A nonuniform fibre grating of length  $L$  is divided into  $M$  uniform gratings, i. e. section, as illustrated in Fig. 1. The propagation through each uniform section  $i$  is described by a matrix  $F_i$  defined such that

$$\begin{bmatrix} R_i \\ S_i \end{bmatrix} = F_i \begin{bmatrix} R_{i-1} \\ S_{i-1} \end{bmatrix}. \quad (2)$$

The matrix  $F_i$  for one section is defined by [1]

$$F_i = \begin{bmatrix} \cosh(\gamma_i \Delta z_i) - j \frac{\hat{\sigma}_i}{\gamma_i} \sinh(\gamma_i \Delta z_i) & -j \frac{\kappa_i}{\gamma_i} \sinh(\gamma_i \Delta z_i) \\ j \frac{\kappa_i}{\gamma_i} \sinh(\gamma_i \Delta z_i) & \cosh(\gamma_i \Delta z_i) + j \frac{\hat{\sigma}_i}{\gamma_i} \sinh(\gamma_i \Delta z_i) \end{bmatrix} \quad (3)$$

here  $j = \sqrt{-1}$ ,  $R_i$  and  $S_i$  are the slowly varying amplitudes of the fundamental mode travelling in the  $+z$  and  $-z$  directions, respectively.  $\gamma_i$  is the coupling coefficient defined as  $\gamma_i \equiv \sqrt{\kappa_i^2 - \hat{\sigma}_i^2}$ , where  $\hat{\sigma}_i = \pi \left( \frac{2n_{eff}}{\lambda} - \frac{1}{\Lambda_i} \right)$ ,  $\kappa_i = \frac{\pi}{\lambda} \Delta n_i$ , and  $\lambda$  is the wavelength, and  $\Delta z_i$  is the section thickness.

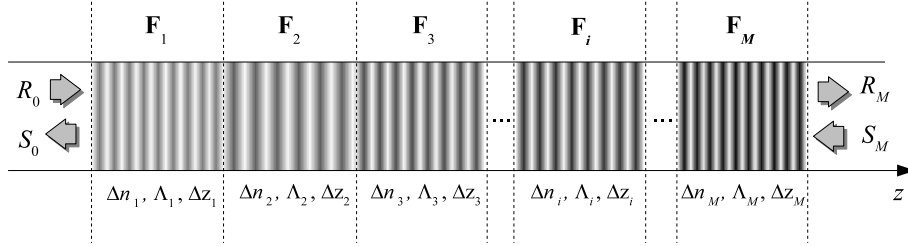


Fig. 1. The transfer matrix method applied to obtain the spectral characteristics of a fibre grating.

Once all the matrices for the individual sections are known, we find the output amplitudes from

$$\begin{bmatrix} R_M \\ S_M \end{bmatrix} = F \begin{bmatrix} R_0 \\ S_0 \end{bmatrix}; \quad F = F_M \cdot F_{M-1} \cdot \dots \cdot F_i \cdot \dots \cdot F_1. \quad (4)$$

The reflection coefficient of the entire grating is defined as  $\rho = S_0/R_0$  and the reflectivity as  $\Gamma = |\rho|^2$  [1, 2]. The main drawback of this method is that  $M$  may not be made arbitrarily large, since the coupled-mode theory approximations are not valid when uniform grating section is only few grating periods long [10]. Thus, it requires  $\Delta z \gg \Lambda$ .

### 3. Grating design using genetic algorithm

A genetic algorithm is a search procedure based on the mechanics of natural selection and natural genetics. The basic structure of a genetic algorithm consists of: a set of *chromosomes*, a *mutation* function, and a *crossover* operator.

A chromosome in the genetic algorithm context is an encoding representation of a solution. The aim of the genetic algorithm is to produce near-optimal solutions by letting a population of random solutions – the *chromosomes*, be manipulated by a sequence of unary and binary transformations – mutation and crossover, respectively. The whole process is governed by a biased *selection* scheme which leads the set of solutions towards high-quality solutions.

The objective of the crossover operator is to implicitly combine good properties of two different chromosomes chosen by the selection operator, and therefore hopefully transfer these properties to their offsprings. Whereas the mutation is to push the search process towards a more wide search area by randomly slightly changing a chromosome encoding so as to increase the chances of finding good solutions.

The process of performing the crossover, the mutation and selecting the survivor elements of the population is then repeated until the stopping criteria are met [11].

In the following section we will show how we have mapped the fibre grating design problem into the genetic algorithm structures.

#### 3.1. Encoding the problem into a chromosome

In general, the design of fibre gratings consists on finding the right values for a set of physical parameters given the desired reflection or transmission spectra. The grating parameters usually are the grating length,  $L$ , the number of sections,  $M$ , the section length,  $\Delta z$ , the section period,  $\Lambda$ , and the modulation amplitude  $\Delta n$ .

Before starting any operation, it is necessary to define the parameters bounds and their representation on the genetic algorithm. For sake of simplicity, we have decided to use only  $\Delta n$  and  $\Lambda$

from  $M$  sections, i. e.,  $2M$  optimisation parameters. Figure 2 shows the encoding chromosome and individuals.

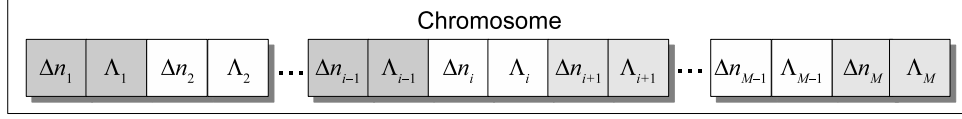


Fig. 2. The encoded chromosome: a float-point vector of amplitude modulations and section periods.

### 3.1.1. The objective function

Genetic algorithm uses the concept of fitness to represent how good is a particular solution. In the fibre grating syntheses problem it is customary to use the inverse of the mean squared error between the computed and the target reflectivity as the *Fitness* function [12], i.e.,

$$Fitness = \left[ \frac{1}{S} \sum_{j=1}^S \left( \frac{\Gamma(\lambda_j) - \Gamma_{target}(\lambda_j)}{\delta\Gamma_j} \right)^2 \right]^{-1}. \quad (5)$$

Where  $\Gamma(\lambda_j)$ ,  $\Gamma_{target}(\lambda_j)$  are the computed and the target reflectivity, respectively, and  $\delta\Gamma_j$  is the tolerance for the  $j^{th}$  wavelength. In the Standard Sampling (SS) technique, the  $\lambda_j$  values are uniformly distributed over  $S = S_{SS}$  samples between  $\lambda_{min}$  and  $\lambda_{max}$  and are given by

$$\lambda_j = \lambda_{min} + (j - 1)\Delta\lambda \quad (6)$$

with  $\Delta\lambda = \frac{\lambda_{max} - \lambda_{min}}{S - 1}$ .

### 3.2. The proposed acceleration technique

It is necessary to compute the *Fitness* of each individual. It means that the processing time of the entire population is affected by the number of wavelength samples,  $S$ . As  $S$  increases, the wavelength sampling presented in Eq. (6) may leads to precise values of the reflectivities. However low values of  $S$  indicate large wavelength spacing, leading to errors on the *Fitness* function. We present in this paper a novel acceleration technique based on the stochastic variation of the wavelength samples.

In our technique, the number of wavelength samples is reduced and the samples are chosen stochastically. Basically, a different set of samples is used to evaluate all individuals of the same generation during the selection process. Thus, after  $N_G$  generations, it is possible that  $N_G$  different set of points has been tested. After the population calculation the best individual is evaluated using the standard sampling technique, i.e.  $S_{SS}$  points, and it is stored. This individual will have samples enough to obtain a reasonable estimation between the target and the calculated curves. It is used as elitist and its fitness is compared with the fitness of the best individual stored from past generation.

In the Reduced Stochastic Sampling (RSS) technique, the  $j^{th}$  wavelength is defined as:

$$\lambda_j = \lambda_{min} + (j - 1)\Delta\lambda + \xi\Delta\lambda \quad (7)$$

where  $\xi$  is a random variable with uniform distribution in  $[0, 1]$ ,  $\Delta\lambda = \frac{\lambda_{max} - \lambda_{min}}{S_{RSS} - 1}$ , and  $S_{RSS}$  is the number of samples in the RSS technique.

It is possible to estimate how the RSS technique improves de GA performance. For a given population size  $N_P$ , traditional GA computes the *Fitness* function  $N_P$  times using  $S_{SS}$  samples. On the other hand, the RSS technique reduces the samples to  $S_{RSS}$  plus the evaluation of the best individuals with  $S_{SS}$  samples. Therefore, one can define a speedup factor  $f$  as

$$f = \frac{S_{SS}N_P}{S_{SS} + S_{RSS}N_P}. \quad (8)$$

#### 4. Numerical examples

To demonstrate the effectiveness of the RSS technique, the design of two fibre Bragg gratings were investigated. All computation performed in this paper have used a AMD *Athlon*<sup>TM</sup> 64 3200+ computer with 1GB of ROM.

The first design consists of a 1 cm long grating with a bandwidth (BW) of 0.4 nm, design wavelength at 1550 nm,  $\Gamma = 1$  inside the BW and  $\Gamma = 0$  otherwise. The number of sections was fixed to  $M = 50$ . The effective refractive index of the fibre core was 1.45. The imposed constrain was the amplitude modulation  $0 \leq \Delta n \leq 8 \times 10^{-4}$ .

Figure 3 shows the *Fitness* curves of best invidual as a function of the processing time for both the standard sampling (SS) and the reduced stochastic sampling (RSS). As one can see, with the RSS technique it is possible to get higher values to the *Fitness* with less computational effort. In Fig. 3 we have chosen  $S_{SS} = 500$ ,  $S_{RSS} = 20$ , and  $N_P = 50$ , resulting in a speedup factor  $f = 16.67$ .

The speedup factor described in Eq. (8) was also tested for others  $S_{RSS}$  values. Figure 4 shows  $f$  as a function of the ratio  $S_{RSS}/S_{SS}$  when  $N_P = 50$  for the previous design. The stars represent the results obtained with the RSS technique. The results show a good agreement between theory and simulations. Figure 5 shows the computed reflectivity spectra obtained

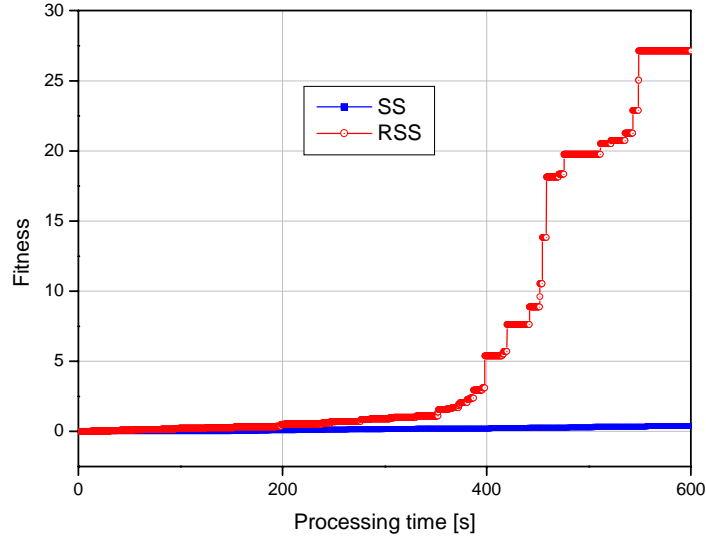


Fig. 3. *Fitness* for standard sampling (SS) and for reduced stochastic sampling (RSS) as a function of the processing time.

with both techniques. The side lobes obtained for the design with the RSS techniques are about

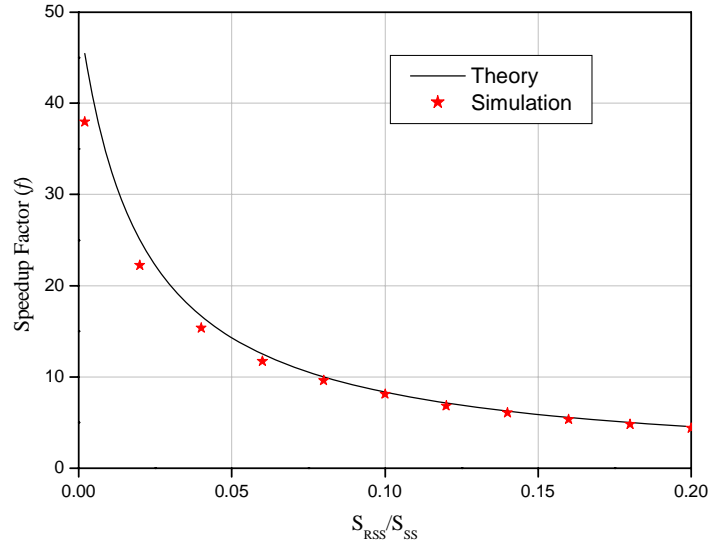


Fig. 4. Speedup factor as a function of the ratio  $S_{RSS}/S_{SS}$  when  $N_P = 50$ . The stars represent the results obtained with the RSS technique.

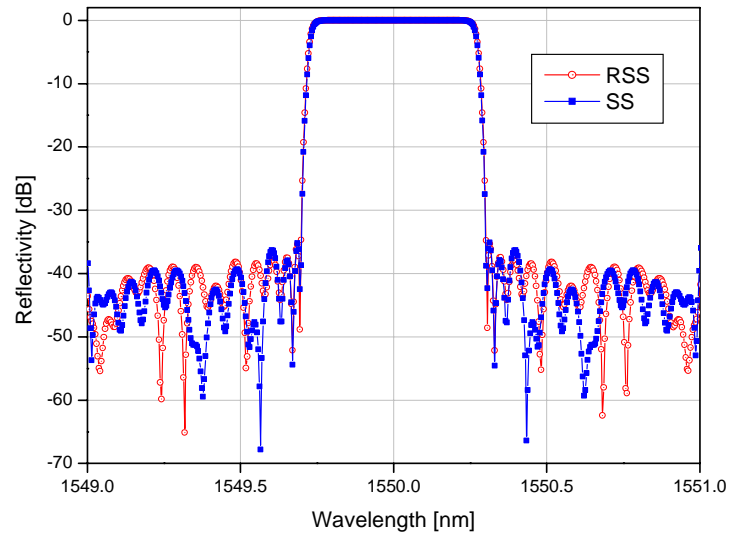
37 dB down. Figure 6 shows the profile of the amplitude modulation  $\Delta n$ . As one can see, the results are very close each other.

The second design consists of a 1 cm long Triangular Fibre Bragg Grating (TFBG) commonly used as a readout device in fibre Bragg grating based sensor applications [13]. The goal of the TFBG filter design is to find the optimum refractive index modulation amplitude,  $\Delta n$ , and section period,  $\Lambda$ , for a linear edge reflectivity spectrum and desired bandwidth.

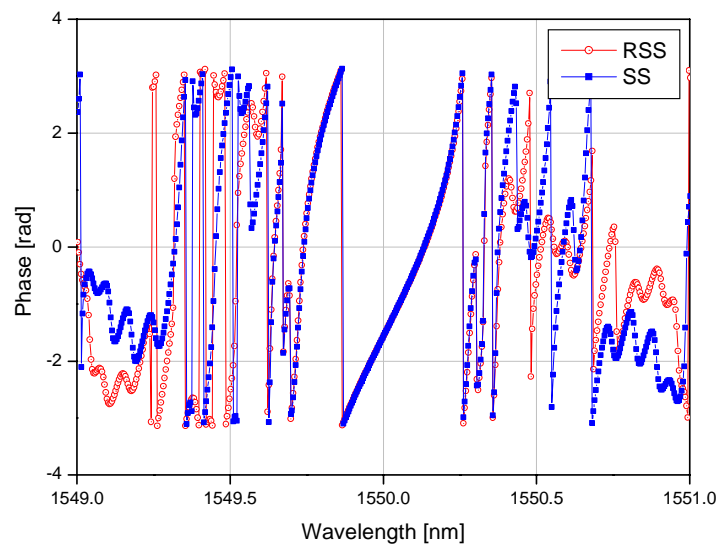
The triangular spectrum mask with 1.0 nm bandwidth and 100% reflectivity at  $\lambda = 1550 \text{ nm}$  is shown in Fig. 7. The number of sections was fixed to  $M = 50$  and the effective refractive index of the fibre core was 1.45. The imposed constrain was the amplitude modulation  $0 \leq \Delta n \leq 8 \times 10^{-4}$ . Figure 7 also shows the computed reflectivities for both SS and RSS techniques. Again, the results are very close to each other.

Figure 8 shows the *Fitness* curves of best individual as a function of the processing time for both techniques. The RSS technique was able to perform better than SS, after 13 minutes of calculations, the RSS *Fitness* was 52% superior.

Figure 9 shows the amplitude modulation and section period profiles for the best individual using the RSS technique. Similar profiles were found using the SS technique.

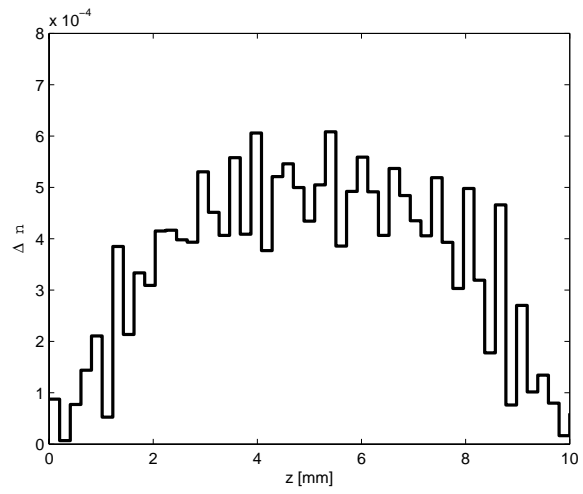


(a)

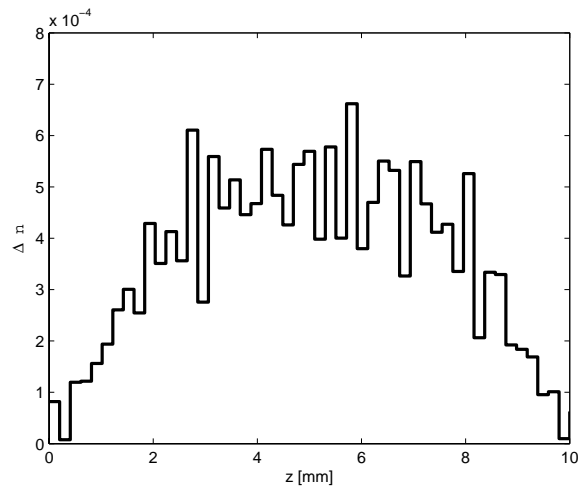


(b)

Fig. 5. Reflectivity curves obtained via standard sampling (SS) and reduced stochastic sampling (RSS). The dotted line represents the SS technique. (a) Reflectivity and (b) phase response.



(a)



(b)

Fig. 6. Profile of the perturbation to the effective refractive index. (a) SS and (b) RSS techniques.



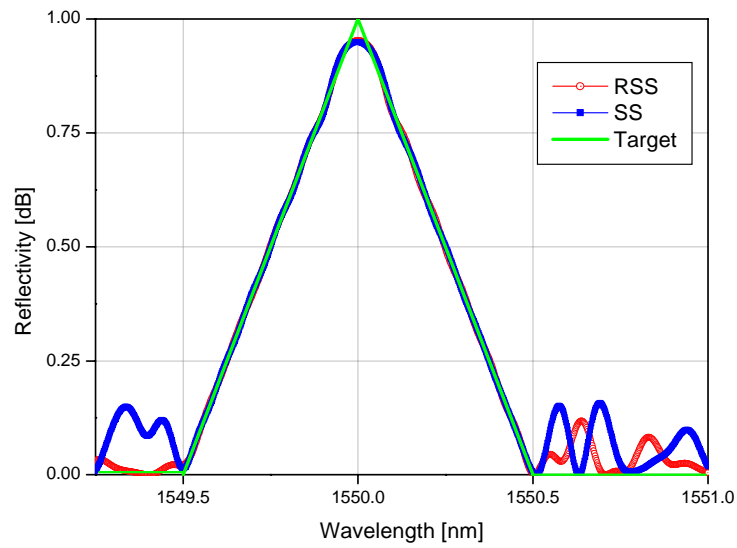


Fig. 7. Target and computed reflectivities using both SS and RSS techniques of a TFBG.

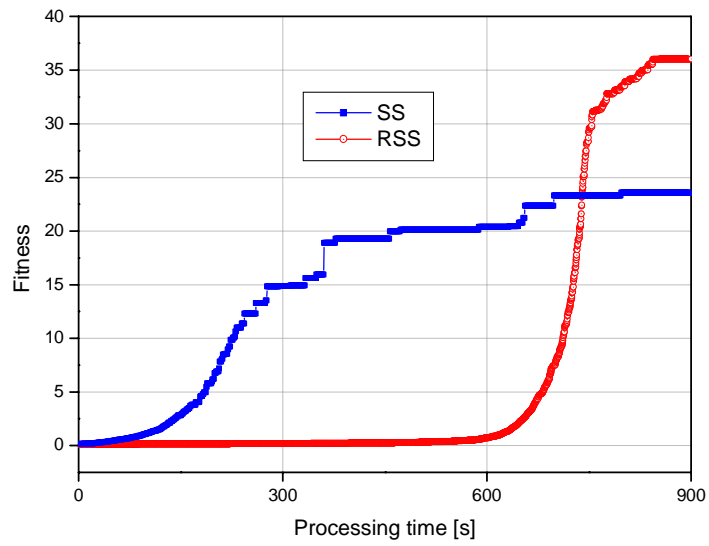
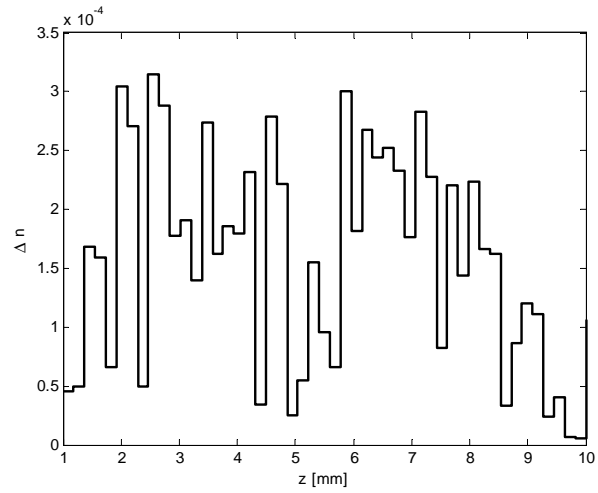
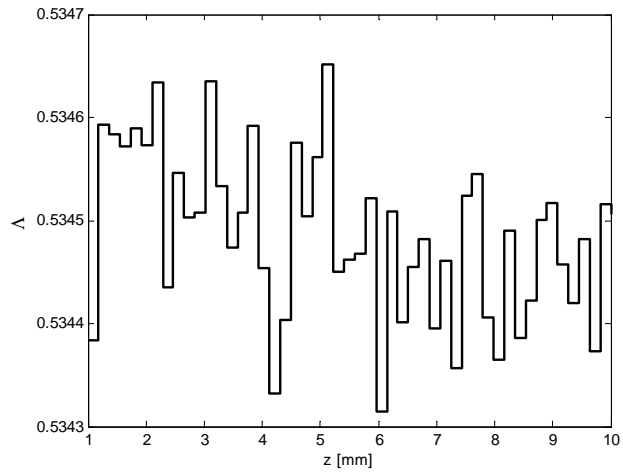


Fig. 8. *Fitness* for standard sampling (SS) and for reduced stochastic sampling (RSS) as a function of the processing time.



(a)



(b)

Fig. 9. (a) amplitude modulation and (b) section period profiles of the best individual using the RSS technique.

## **5. Conclusions**

In this paper we have presented a new acceleration technique for the design of fibre gratings using genetic algorithm. Our technique is based on the stochastic variation of the wavelength samples and it is able to significantly reduce the computation effort. The numerical examples presented show that the RSS technique can be used with a minimum of wavelength samples.

We also have tested both techniques for the design of different grating profiles. In all tests RSS have shown the same performance. In addition, the proposed method is general, and would thus be useful for other inverse problems such as multilayer filters, thin film devices and photonic crystals based devices.

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